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Assessment Answer

$$\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t \quad \ldots \quad eqn (1)$$

when $t=0$ $x=0.1$ $\frac{dx}{dt}=0$

Using Auxiliary equation method

$$\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = 0 \quad \ldots \quad equation (2)$$

if $x = Ae^{kt}$

$\frac{dx}{dt} = kAe^{kt}$ $\quad \frac{dx}{dt} = kx$

$$\frac{d^2x}{dt^2} = k^2 Ae^{kt}$$

$$\frac{d^2x}{dt^2} = k^2 x$$

Sub $\frac{d^2x}{dt^2}$ and $\frac{dx}{dt}$ into eqn (2)

$k^2x + 5kx + 6x = 0$

dividing through by x

$k^2 + 5k + 6 = 0$ $\longrightarrow$ Auxiliary Equation

$k^2 + 2k + 3k + 6 = 0$

$k(k+2) + 3(k+2) = 0$

$(k+2)(k+3) = 0$

$k = -2$ or $k = -3$

Hence

Since $y = Ae^{k_1x} + Be^{k_2x}$

$\therefore$ $x = Ae^{-2t} + Be^{-3t}$ $\longrightarrow$ complementary function.

for $\cos t$

$x = A\cos t + B\sin t$

$\frac{dx}{dt} = -A\sin t + B\cos t$

$\frac{d^2x}{dt^2} = -A\cos t - B\sin t$

from eqn (1)

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Substitute $\frac{d^2x}{dt^2} = -A\cos t - B\sin t$ and $\frac{dx}{dt} = -A\sin t + B\cos t$ and

$x = A\cos t + B\sin t$ into eqn (1)

$= (-A\cos t - B\sin t) + 5(-A\sin t + B\cos t) + 6(A\cos t + B\sin t) = \cos t$

$= -A\cos t - B\sin t - 5A\sin t + 5B\cos t + 6A\cos t + 6B\sin t = \cos t$

$= \cos t(-A + 5B + 6A) + \sin t(-B + 6B - 5A) = \cos t$

$-A + 5B + 6A = 1\cos t$

$-5A + 6B - B = \cos\sin t$

$\therefore -A + 5B + 6A = 1$

$-5A + 6B - B = 0$

$5A + 5B = 1 \quad \cdots$ (iii)

$-5A + 5B = 0 \quad \cdots$ (iv)

$10A = 1$

$A = \frac{1}{10}$

Subst $A = 1/10$ into eqn (iii)

$5(1/10) + 5(B) = 1$

$1/2 + 5B = 1$

$5B = 1 - 1/2$

$5B = 1/2$

$B = \frac{1/2}{5}$

$B = 1/10$.

$x = 1/10 \cos t + 1/10 \sin t \rightarrow$ Partial Integral (P.I)

$GS = C.F + P.I$

$x = Ae^{-2t} + Be^{-3t} + 1/10 \cos t + 1/10 \sin t$

$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - \frac{\sin t}{10} + \frac{1}{10}\cos t$

where $x = 0.1$ $dx/dt = 0$ $t = 0$

$0.1 = Ae^{-2(0)} + Be^{-3(0)} + 1/10 \cos(0) + 1/10 \sin(0)$

$0.1 = A + B + 1/10 + 0$

$0.1 - 1/10 = A + B$

$A + B = 0 \quad \cdots$ eqn (v)

also

$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} - 1/10 \sin(0) + 1/10 \cos(0)$

$0 = -2A - 3B + 1/10$

$-1/10 = -2A - 3B \quad \cdots$ (vi)

Simultaneously

$A + B = 0 \quad$ (v)

$-2A - 3B = -1/10 \quad \cdots$ (vi)

$A = -B \quad \cdots$ eqn (vii)

subst eqn (vii) into eqn (vi)

$-2(-B) - 3B = -1/10$

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$-2B \quad -3B = -1/10$

$-B = -1/10$

$B = 1/10$

$A = -1/10$

$$x = -\tfrac{1}{10}e^{-2t} + \tfrac{1}{10}e^{-3t} + \tfrac{1}{10}\cos t + \tfrac{1}{10}\sin t$$

$$x = \frac{1}{10}\left(-e^{-2t} + e^{-3t} + \cos t + \sin t\right)$$

from the graph amplitude $k = 0.41$

Period $(T) = 14.9 - 8.7 = 6.2$ secs

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{6.2} = 0.1 \text{ rad/s or } \pi/3 \text{ rad}$$

a = the value of t for which $\sin t = \cos t$

$a = 45^\circ$ or $\pi/4$ rad

In the form $k \sin(t + a)$, the steady state solution is given as

$$0.141 \sin(t + 45^\circ)$$