

5/10/18

9. Using Auxiliary method
$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

$$m^2 + 5m + 6 = 0$$

$$m^2 + 2m + 3m + 6 = 0$$

$$m(m+2) + 3(m+2) = 0$$

$$(m+2)(m+3) = 0$$

$$m_1 = -2, m_2 = -3$$

$$x = Ae^{-2t} + Be^{-3t} \quad (\text{Complementary function})$$

P.I : $f(x) = \cos t$

$$x = C\cos t + D\sin t$$

$$\frac{dx}{dt} = -C\sin t + D\cos t$$

$$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$$

Putting back into the original equation

$$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$$

$$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$$

$$\cos t(-C + 5D + 6C) + \sin t(-D - 5C + 6D) = \cos t$$

$$-C + 5D + 6C = 1$$

$$5D + 5C = 1 \quad \dots (i)$$

$$-D - 5C + 6D = 0$$

$$-5C + 5D = 0$$

$$5D - 5C = 0 \quad \dots (ii)$$

$$5D = 5C$$

$$D = \frac{5C}{5}, \quad D = C$$

$D = -C$ into equ (i)

$$5(C) + 5C = 1$$

$$10C = 1$$

$$C = \frac{1}{10}$$

$$D = \frac{1}{10}$$

$$G.S = C.F + P.I$$

$$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$x = 0.1, t = 0, \frac{dx}{dt} = 0$$

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10} \cos(0) + \frac{1}{10} \sin(0)$$

$$0.1 = A(1) + B(1) + \frac{1}{10}(1)$$

$$0.1 = A + B + \frac{1}{10}$$

$$0.1 - \frac{1}{10} = A + B$$

$$0 = A + B \quad \dots (i)$$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - \frac{1}{10} \sin t + \frac{1}{10} \cos t$$

$$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} - \frac{1}{10} \sin(0) + \frac{1}{10} \cos(0)$$

$$= -2A(1) - 3B(1) + \frac{1}{10}(1)$$

$$0 = -2A - 3B + \frac{1}{10} \quad \dots (ii)$$

$$A = -B$$

$$-2(-B) =$$

$$2A + 2B = \frac{1}{10}$$

$$2(-B) + 2B = \frac{1}{10}$$

$$-2B + 2B = \frac{1}{10}$$

$$B = \frac{1}{10}$$

$$A = -\frac{1}{10}$$

P.S

$$x = -\frac{1}{10} e^{-2t} + \frac{1}{10} e^{-3t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$c. \quad x = -0.1e^{-2t} + 0.1e^{-3t} + 0.1\sin t + 0.1\cos t$$

at steady state

$$t \rightarrow \infty$$

$$\therefore 0.1e^{-2t} \Rightarrow 0 \text{ and } 0.1e^{-3t} \Rightarrow 0$$

$$x = 0.1[\sin t + \cos t]$$

$$= 0.1\sin t + 0.1\cos t$$

$$x = K\sin(t + a)$$

$K\sin t \cos a + K\sin a \cos t$ for trigonometry

$$K\sin t \cos a + K\sin a \cos t = 0.1\sin t + 0.1\cos t$$

$$K\cos a = 0.1 \text{ and } K\sin a = 0.1$$

$$K^2\sin^2 a + K^2\cos^2 a = 0.1^2 + 0.1^2$$

$$K^2(\sin^2 a + \cos^2 a) = 0.02$$

$$\text{since } \sin^2 a + \cos^2 a = 1$$

The equation becomes

$$K^2 = 0.02$$

$$\therefore K = \sqrt{0.02} = 0.1414$$

$$\text{and } K\sin a = K\cos a$$

$$\therefore \sin a = \cos a$$

$$\frac{\sin a}{\cos a} = 1, \quad \frac{\sin}{\cos} = \tan$$

$$\tan a = 1$$

$$a = \tan^{-1}$$

$$a = 45^\circ$$

$$\therefore x = 0.1(\sin t + \cos t) = K\sin(t + a) \\ = 0.1414\sin(t + 45^\circ)$$

$$x = 0.1414\sin(t + 45^\circ)$$