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$$(1) m^2 + 5m + 6 = 0$$

$$m^2 + 3m + 2m + 6 = 0$$

$$m(m+3) + 2(m+3) = 0$$

$$(m+2)(m+3) = 0$$

$$m_1 = -2 \text{ and } m_2 = -3$$

$$\text{P.I: } x = C \cos t + D \sin t$$

$$\text{C.F: } x = A e^{-2t} + B e^{-3t}$$

$$x = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

$$-C \cos t - D \sin t - 5(C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t$$

$$-C \cos t - D \sin t - 5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t = \cos t$$

$$-C \cos t + 5D \cos t + 6C \cos t - D \sin t - 5C \sin t + 6D \sin t = \cos t$$

$$\cos t (-C + 5D + 6C) + \sin t (-D - 5C + 6D) = \cos t$$

$$\cos t (5D + 5C) + \sin t (-5C + 5D) = \cos t$$

$$5D + 5C = 1 \quad \dots (1)$$

$$-5C + 5D = 0 \quad \dots (2)$$

$$5D = 5C$$

$$D = C$$

$$5D + 5C = 1$$

$$5D + 5(D) = 1$$

$$10D = 1$$

$$D = \frac{1}{10}$$

$$\therefore C = \frac{1}{10}$$

P.I - partial integral

$$x = \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

General solution = C.F + P.I

$$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - \frac{\sin t}{10} + \frac{\cos t}{10}$$

when $x = 0.1$

$$\frac{dx}{dt} = 0 \text{ and } t = 0$$

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10} \cos(0) + \frac{1}{10} \sin(0) \quad (2)$$

$$0.1 = A + B + \frac{1}{10} + 0 \quad (3)$$

$$0.1 - \frac{1}{10} = A + B$$

$$0.1 - 0.1 = A + B$$

$$0 = A + B \quad \dots (3)$$

$$A = -B$$

Also,

$$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} - \frac{\sin(0)}{10} + \frac{\cos(0)}{10}$$

$$0 = 2A - 3B + \frac{1}{10}$$

$$-\frac{1}{10} = -2A - 3B \quad \dots (4)$$

Solving equations (3) and (4) simultaneously

$$A + B = 0$$

$$-2A - 3B = -\frac{1}{10}$$

$$A = -B$$

$$-2(-B) - 3B = -\frac{1}{10}$$

$$2B - 3B = -\frac{1}{10}$$

$$-B = -\frac{1}{10} \quad B = \frac{1}{10}$$

$$A = -B$$

$$A = -\left(\frac{1}{10}\right)$$

$$A = -\frac{1}{10}$$

$$x = -\frac{1}{10} e^{-2t} + \frac{1}{10} e^{-3t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

(2) Done on matlab

(3) From the graph plotted on matlab

Amplitude, $K = 0.14$

$$A \sin(\omega t + \phi)$$

$$\text{Period, } T = 8.55 - 2.40 = 6.15$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{6.15}$$

$$= 1 \text{ rad/s or } \frac{\pi}{2} \text{ rad}$$

a = the value of t for which $\sin t = \cos t$

$$a = 45^\circ \text{ or } \frac{\pi}{4} \text{ rad}$$

Comparing with $K \sin(t + a)$

The steady state solution is given as

$$0.14 \sin(t + 45^\circ)$$

or

$$\frac{\sqrt{2}}{10} \sin(t + \frac{\pi}{4})$$