ts ▸ MATLAB

ditor - C:\Users\OCHENI VICTOR\Documents\MATLAB\mid.m

mid.m × +

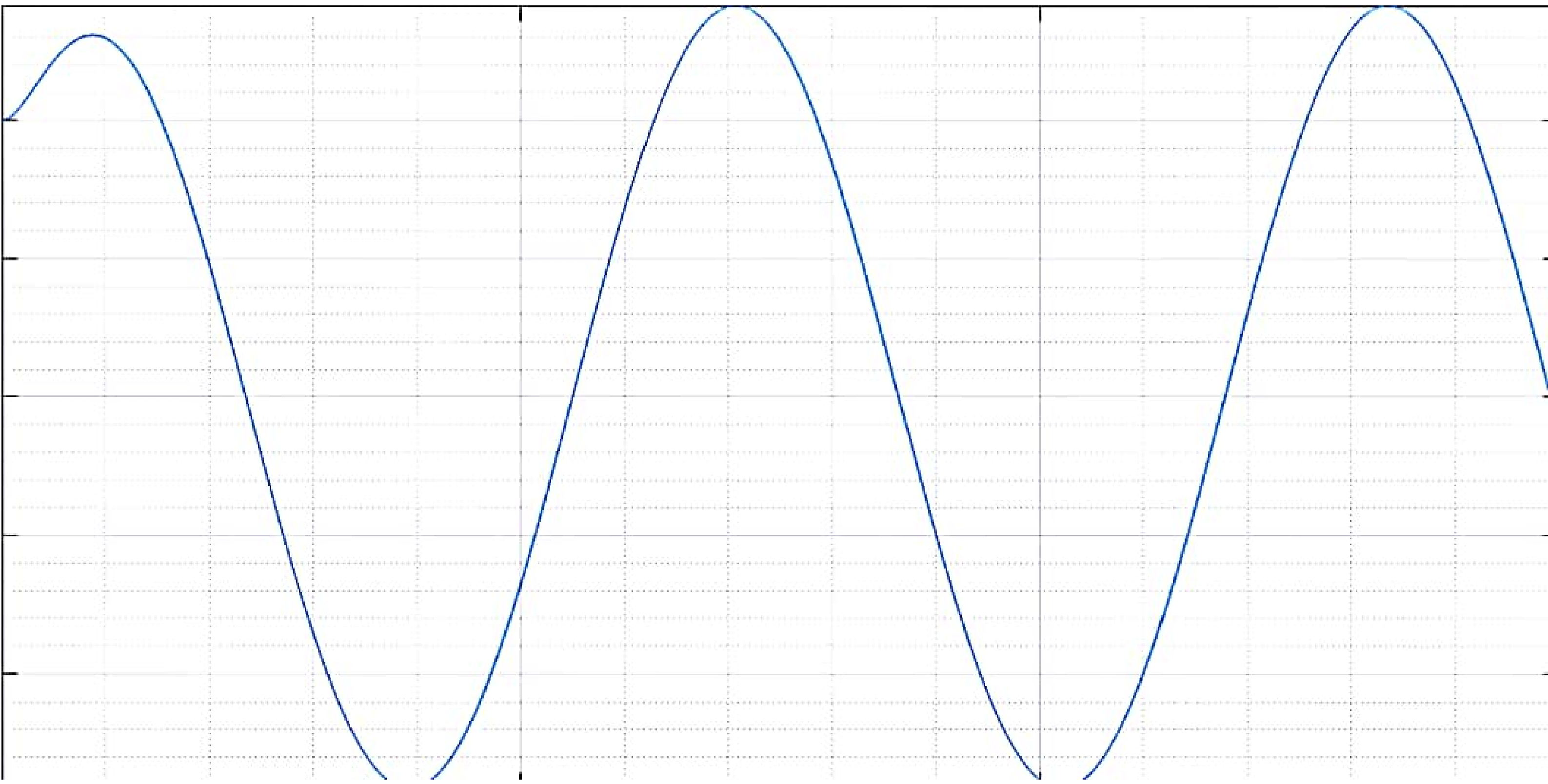
```
commandwindow
clear
clc
close all
syms t
x =(0.1)*(-exp(-2*t)+exp(-3*t)+cos(1*t)+sin(1*t))
ts=[0:0.01:15]
xs= subs(x,ts)
figure (1)
plot(ts,xs)
xlabel('Time(seconds)')
ylabel('Vibrations')
grid on
grid minor
axis tight
```

ommand Window

```
    14.9400    14.9500    14.9600    14.9700    14.9800    14.9900    15.0000


xs =

[ 1/10, cos(1/100)/10 - exp(-1/50)/10 + exp(-3/100)/10 + sin(1/100)/10, cos(1/50)/10 - exp(-1/25)/10 + exp(-3/50)/10 + sir
>>
```



Victor Ocheni
16/ENG04/087
Electrical Electronics
ENG 381

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Using the Auxiliary Equation method to obtain to solution of the model.

1. Find the complementary function CF

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$$

Recall: $\frac{d^2x}{dt^2} = k^2x$

$\frac{dx}{dt^2} = kx$

$k^2 + 5k + 6 = 0$

$k^2 + 2k + 3k + 6 = 0$

$k(k+2) + 3(k+2) = 0$

$(k+2)(k+3) = 0$

$k+2 = 0$ or $k+3 = 0$

$k_1 = -2$ and $k_2 = -3$

Therefore $x = Ae^{k_1t} + Be^{k_2t}$

$x = Ae^{-2t} + Be^{-3t}$

2. For the particular Integral

$f(t) = \cos t$

$x = C\cos t + D\sin t$

$\therefore \frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

Substituting into the original equ.

$-C\cos t - D\sin t + 5[-C\sin t + D\cos t] + 6[C\cos t + D\sin t] = \cos t$

$-C\cos - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = D\sin t - 5C\sin t$

$= \cos t.$

$5D\cos t + 5C\cos t + 5D\sin t - 5C\sin t = \cos t$

$\cos t\,[5D + 5C] + \sin t\,[5D - 5C] = \cos t$

Equating coefficient

$5D + 5C = 1$

$5D - 5C = 0$

$5C - [-5C] = 1 - 0$

$5C + 5C = 1$

$10C = 1$

$C = 1/10$

$\therefore C = 1/10$

Sub into eqn (11)

$5D + 5\left[1/10\right] = 1.$

$5D + \frac{1}{2} = 1$

$5D + \frac{1}{2} = 1$

$5D = \frac{1}{1} - \frac{1}{2}$

$5D = \frac{1}{2}$

$10D = 1$

$D = 1/10$

$\therefore$ particular integral is

$x = \frac{\cos t}{10} + \frac{\sin t}{10}$

Complete general solution = CF + PI

Solution of the model is $x = Ae^{-2t} + Be^{-3t} + \frac{\cos t}{10} + \frac{\sin t}{10}$

at $t = 0$, $x = 0.1$ and $\frac{dx}{dt} = 0$

$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{\cos(0)}{10} + \frac{\sin(0)}{10}$

$0.1 = A + B + \frac{1}{10} + 0$

$A + B = 0.1 - \frac{1}{10} = \frac{1}{10} - \frac{1}{10} = 0 \quad = A + B = 0$

$$x = Ae^{-2t} + Be^{-3t} + \frac{\cos t}{10} + \frac{\sin t}{10}$$

$$\frac{dx}{dt} = -2Ae^{-2t} - Be^{-3t} - \frac{\sin t}{10} + \frac{\cos t}{10}$$

$$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} - \frac{\sin(0)}{10} + \frac{\cos(0)}{10}$$

$$= -2A - 3B - 0 + \frac{1}{10}$$

$$2A + 3B = \frac{1}{10} \quad \cdots (iv)$$

$$A + B = 0 \quad \cdots (iii)$$

$$2A + 3B = \frac{1}{10} \quad \cdots (v)$$

From eqn (iii), $A + B = 0$

$$A = -B$$

Sub. eqn (v) into (iv)

$$2(-B) + 3B = \frac{1}{10}$$

$$-2B + 3B = \frac{1}{10}$$

$$B = \frac{1}{10}$$

Sub B into eqn (iii)

$$A + \left[\frac{1}{10}\right] = 0$$

$$A = -\frac{1}{10}$$

The particular solution for the model is

$$x = -\frac{e^{-2t}}{10} + \frac{e^{-3t}}{10} + \frac{\cos t}{10} + \frac{\sin t}{10}$$

$$x = \frac{1}{10}\left[-e^{-2t} + e^{-3t} + \cos t + \sin t\right].$$

III Write a steady-state solution of the system in form of $x = k\sin(t + \varphi)$

From the equation

$$x = Ae^{-2x} + Be^{-3x} + \frac{1}{10}(\sin t + \cos t)$$

The equation has a changing or transcient part and a steady part

The transcient part denoted as $e^{k_1 t}$ and $k_2 t$

The steady part denoted $1/10(\sin t + \cos t)$

In trigonometric identities

$$A\sin wt + B\cos wt = c\sin(t + \theta)$$

where $\theta = \tan^{-1}(A/B)$

$$c = (A^2 + B^2)^{1/2}$$

steady part becomes

$$\frac{1}{10}\sin t + \frac{1}{10}\cos t = k\sin(t + \varphi)$$

where

$$k = ((0.1)^2 + 0.1^2)^{1/2} = \frac{\sqrt{2}}{10}$$

$$\varphi = \tan^{-1}\left(\frac{0.1}{-0.1}\right.$$

$$\varphi = -45^0$$

$$k\sin(t + \varphi) = \frac{\sqrt{2}}{10}\sin(t - 45)$$

$\frac{\sqrt{2}}{10}\sin(t - 45)$ is the steady state equation.