IGBOEKWE KENENNA CHIDIEBERE
CIVIL ENGINEERING
16/ENG03/031
ENG 381

1) $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

assuming $\cos t = 0$

$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$

$m^2 + 5m + 6 = 0$

$m^2 + 3m + 2m + 6 = 0$

$m_1 = -2$ or $m_2 = -3$

C.F $\Rightarrow x = Ae^{-2t} + Be^{-3t}$

P.I $\Rightarrow x = C\cos t + D\sin t$

$\frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

$\cos t(-C + 5D + 6C) + \sin t(-D - 5C + 6D) = \cos t$

$\cos t(5C + 5D) + \sin t(5D - 5C) = \cos t$

$\cos t(5C + 5D) = \cos t$

$5C + 5D = 1$ ——①

$\sin t(5D - 5C) = 0$

$5D - 5C = 0$ ——②

Solve equ ① and ② simultaneously

$10D = 1$

$D = 1/10$

Substitute $1/10$ for D in equ(1)

$5C + 5D = 1$

$$\frac{5}{s^2} + \frac{5}{s}\left(\frac{1}{10}\right) = 1$$

$$\frac{5}{s^2} + \frac{5}{10s} = 1$$

$$\frac{5}{s^2} + \frac{1}{2s} = 1$$

$$\frac{5}{s^2} = 1 - \frac{1}{2s}$$

$$C = \frac{1}{10}$$

$$P.I \Rightarrow x = \frac{1}{10}\cos t + \frac{1}{10}\sin t$$

$$G.S = C.F + P.I$$

$$G.S \Rightarrow x = Ae^{-2t} + Be^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$$

when $t = 0$, $x = 0.1$, $\frac{dx}{dt} = 0$

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10}\cos 0 + \frac{1}{10}\sin 0$$

$$0.1 = A + B + \frac{1}{10}$$

$$A + B = 0.1 - \frac{1}{10}$$

$$A + B = 0$$

$$A = -B \quad \ldots (3)$$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - \frac{1}{10}\sin t + \frac{1}{10}\cos t$$

$$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} - \frac{1}{10}\sin 0 + \frac{1}{10}\cos 0$$

$$0 = -2A - 3B + 0.1$$

$$2A + 3B = 0.1 \quad \ldots (4)$$

Sub. equ. (3) into equ (4)

$$2(-B) + 3B = 0.1$$

$$-2B + 3B = 0.1$$

$$B = 0.1$$

$A = -0.1$

P.S $\Rightarrow$ $x = 0.1e^{-2t} + 0.1e^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

ii) MATLAB Program:

Command Window

```
clear
clc
close all
syms t
x(t) = 0.1 * (exp(-3*t)) - (exp(-2*t)) + cos(t) + sin(t))
tn = [0:0.01:15]
xd = subs(x(t), tn)
figure 1
Plot(tn, xd)
axis tight
grid on
grid minor
```

iii) Steady state solution in form of $x = K\sin(t+a)$

Using the P.I from the general solution: $x = C\cos t + D\sin t$

Assume $C = K\cos a$ and $D = K\sin a$

$x = (K\cos a)\sin t + (K\sin a)\cos t$

$x = K[\sin t\cos a + \cos t\sin a]$

$x = K[\sin t\cos a - \cos t(-\sin a)]$

$x = K[\cos(\frac{\pi}{2} - t)\cos(-a) - \sin(\frac{\pi}{2} - t)\sin(-a)]$

$x = K[\cos((\frac{\pi}{2} - t) + (-a))]$

$x = K[\cos((\frac{\pi}{2} - t) - a)]$

$x = K[\cos\frac{\pi}{2} - (t+a)]$

$x = K[\cos\frac{\pi}{2}\cos(t+a) + \sin\pi/2\sin(t+a)]$

$x = K[0 \times \cos(t+a) + 1 \times \sin(t+a)]$

$x = K\sin(t+a)$.