from Equation ② $5c - 5D = 0$

③ $5c + 5D = 1$

Equation ③ - ② $10D = 1$

$D = \frac{1}{10}$

Substitute $D = \frac{1}{10}$ into Eqn ②

$5c - 5\left(\frac{1}{10}\right) = 0$

$5c - \frac{1}{2} = 0$

$5c = \frac{1}{2}$

$c = \frac{1}{10}$

P.I $C\sin t + D\cos t$

$= \frac{1}{10}\sin t + \frac{1}{10}\cos t$

$x = CF + PI = Ae^{-2t} + Be^{-3t} + \frac{1}{10}\sin t + \frac{1}{10}\cos t$

$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10}(\sin t + \cos t)$ --- ④

At $t = 0,\ x = 0.1$

$\frac{dx}{dt} = 0$

$\frac{dx}{dt} = -2Ae^{-2t} + (-3)Be^{-3t} + \frac{1}{10}(\cos t - \sin t)$ --- ⑤

from Eqn ④ $x = 0.1 \quad t = 0$

$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10}(\sin(0) + \cos(0))$

$A + B + \frac{1}{10} = 0.1$

$A + B = 0$ --- ⑥

from Eqn ⑤ $\frac{dx}{dt} = 0, \quad A = 0$

$0 = -2Ae^{-2(0)} - 3e^{-3(0)} + \frac{1}{10}(\cos(0) - \sin(0))$

$= -2A - 3B + \frac{1}{10}$

$2A + 3B = 0.1$ --- ⑦

Multiply Eqn 6 by ②

$2(A+B) = 0$

Equation $2A + 2B = 0$ --- (8)

Eqn (7) - (8) : $2A + 3B = 0.1$

$2A + 2B = 0$

$B = 0.1$

Substitute $B = \frac{1}{10}$ into Eqn (7)

$2A + 2\left(\frac{1}{10}\right) = \frac{1}{10}$

$2A + \frac{1}{5} = \frac{1}{10}$

$2A = \frac{1}{10} - \frac{1}{5} = -\frac{1}{10}$

$A = \frac{-\frac{1}{10}}{2} = -\frac{1}{20}$

$$x = -\frac{1}{20}e^{-2t} + \frac{1}{10}e^{-3t} + \frac{1}{10}(\sin t + \cos t)$$

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$$\frac{d^2x}{dt^2} + 5\frac{dy}{dx} + 6x = \cos t$$

Given that when $t=0$, $x=0.1$ $\quad \frac{dx}{dt}=0$

(i) Using Auxiliary method, obtain the solution in form of an expression having x as a function of t.

$$\frac{dx^2}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t \quad ---(1)$$

$$x = CF + PI$$

CF: $\lambda^2 + 5\lambda + 6 = 0$

$\lambda^2 + 2\lambda + 3\lambda + 6 = 0$

$\lambda(\lambda+2) + 3(\lambda+2) = 0$

$(\lambda+2)(\lambda+3) = 0$

$\lambda_1 = -2 \quad \lambda_2 = -3$

CF: $Ae^{\lambda_1 t} + Be^{\lambda_2 t}$

$Ae^{-2t} + Be^{-3t}$

P.I: $x(t) = \cos t$

$x = C\sin t + D\cos t$

$\frac{dx}{dt} = C\cos t - D\sin t$

$\frac{dx^2}{dt^2} = -C\sin t - D\cos t$

$-C\sin t - D\cos t + 5C\cos t - 5D\sin t + 6C\sin t + 6D\cos t = \cos t$

$(-C - 5D + 6C)\sin t + (-D + 5C + 6D)\cos t = \cos t$

Comparing Coefficient

$5C - 5D = 0 \quad ---(2)$

$5C + D = 1 \quad ---(3)$