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17/ENG01 1033.

ENG 381

$$1) m^2 + 5m + 6 = 0$$

$$m^2 + 3m + 2m + 6 = 0$$

$$m(m+3) + 2(m+3) = 0$$

$$(m+2)(m+3) = 0$$

$$m = -2, m = -3$$

$$P.I: x = C \cos t + D \sin t$$

$$C.F: x = A e^{-2t} + B e^{-3t}$$

$$x = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

$$-C \cos t + D \sin t - 5C \sin t + 5D \cos t + 6(C \cos t + D \sin t) = \cos t$$

$$-C \cos t - D \sin t - 5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t = \cos t$$

$$-C \cos t + 5D \cos t + 6C \cos t - D \sin t - 5C \sin t + 6D \sin t = \cos t$$

$$\cos t (-C + 5D + 6C) + \sin t (-D - 5C + 6D) = \cos t$$

$$\cos t (5D + 5C) + \sin t (-5C + 5D) = \cos t$$

$$5D + 5C = 1$$

$$-5C + 5D = 0$$

$$5D = 5C$$

$$5D + 5C = 1$$

$$5D + 5(D) = 1$$

$$10D = 1$$

$$D = \frac{1}{10}$$

$$\therefore C = \frac{1}{10}$$

P.I - Partial Integral

$$x = \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

General solution = C.f + P.I.

$$x = A e^{-2t} + B e^{-3t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$\frac{dx}{dt} = -2A e^{-2t} - 3B e^{-3t} - \frac{\sin t}{10} + \frac{\cos t}{10}$$

when $x = 0.1$

$$\frac{dx}{dt} = 0 \quad \& \quad t = 0.$$

$$0.1 = A e^{-2(0)} + B e^{-3(0)} + \frac{1}{10} \cos(0) + \frac{1}{10} \sin(0)$$

$$0.1 = A + B + \frac{1}{10} + 0$$

$$0.1 - \frac{1}{10} = A + B$$

$$0.1 - 0.1 = A + B$$

$$0 = A + B \quad \text{--- (3)}$$

$$A = -B$$

Also,

$$0 = -2A e^{-2(0)} - 3B e^{-3(0)} - \frac{\sin(0)}{10} + \frac{\cos(0)}{10}$$

$$0 = 2A - 3B + \frac{1}{10}$$

$$-\frac{1}{10} = -2A - 3B \quad \text{--- (4)}$$

Solving equations (3) & (4) simultaneously.

$$A + B = 0$$

$$-2A - 3B = -\frac{1}{10}$$

$$A = -B$$

$$-2(-B) - 3B = -\frac{1}{10}$$

$$2B - 3B = -\frac{1}{10}$$

$$-B = -\frac{1}{10} \quad ; \quad B = \frac{1}{10}$$

$$\lambda = -B$$

$$A = -\left(\frac{1}{10}\right)$$

$$A = -\frac{1}{10}$$

$$x = -\frac{1}{10} e^{-2t} + \frac{1}{10} e^{-3t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t.$$

3) from the graph plotted on matlab,

Amplitude, $k = 0.14$.

$$A \sin(\omega t + \phi)$$

$$\text{Period, } T = 8.55 - 2.40 = 6.15$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{6.15}$$

$$= 1 \text{ rad/s or } \pi/3 \text{ rad.}$$

$\alpha =$ the value of t for which $\sin t = \cos t$

$$\alpha = 45^\circ \text{ or } \frac{\pi}{4} \text{ rad.}$$

Comparing with $k \sin(t + \alpha)$

The steady state solution is given as

$$0.14 \sin(t + 45^\circ).$$

or

$$\frac{\sqrt{2}}{10} \sin(t + \pi/4)$$