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Assignment

The dynamic model of a body in motion, performing damped forced vibrations is as in equation (1).

$$\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t$$

where $t = 0$; $x = 0.1$ and $\frac{dx}{dt} = 0$.

Solution.

If $\frac{d^2y}{dx^2} + a\frac{dy}{dx} + cy = 0$.

then $y = x$ and $x = t$

Hence:

$$\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t$$

Assume $x = Ae^{kt}$

$k^2x + 5kx + 6x = 0$ — Auxiliary Equation.

$k^2 + 5k + 6 = 0$

$k^2 + 3k + 2k + 6 = 0$

$k(k+3) + 2(k+3) = 0$

$(k+2)(k+3) = 0$

$k = -2$ or $k = -3$.

General solution

$$x = Ae^{k_1 t} + Be^{k_2 t}$$

$$x = Ae^{-2t} + Be^{-3t}$$

For cost

$x = \cos t$

$x = A\cos t + B\sin t$

$\frac{dx}{dt} = -A\sin t + B\cos t$

$\frac{d^2x}{dt^2} = -A\cos t - B\sin t.$

In $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t.$

$(-A\cos t - B\sin t) + 5(-A\sin t + B\cos t) + 6(A\cos t + B\sin t) = \cos t$

$-A\cos t - B\sin t - 5A\sin t + 5B\cos t + 6A\cos t + 6B\sin t = \cos t$

$-A\cos t + 5B\cos t + 6A\cos t - B\sin t - 5A\sin t + 6B\sin t = \cos t$

$\cos t(-A + 5B + 6A) + \sin t(-B - 5A + 6B) = \cos t.$

$\cos t(5A + 5B) + \sin t(-5A + 5B) = \cos t.$

$5A + 5B = 1 \quad \ldots (1)$

$-5A + 5B = 0 \quad \ldots (2)$

$5B = 5A$

$B = A$

$5(B) + 5B = 1$

$10B = 1$

$B = 1/10$; Since $A = B$

$A = 1/10.$

P.I - Partical Integration.

$x = \frac{1}{10}\cos t + \frac{1}{10}\sin t.$

General solution = C.F + P.I

$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t.$

$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - \frac{\sin t}{10} + \frac{\cos t}{10}$

$x = 0.1$; $\frac{dx}{dt} = 0$; $t = 0$.

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10}\cos(0) + \frac{1}{10}\sin(0)$$

$$0.1 = A + B + \frac{1}{10} + 0$$

$$0.1 - \frac{1}{10} = A + B$$

$$0.1 - 0.1 = A + B$$

$$0 = A + B \quad \text{——equ *}$$

Also,

$$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} - \frac{\sin(0)}{10} + \frac{\cos(0)}{10}$$

$$0 = 2A - 3B + \frac{1}{10}$$

$$-\frac{1}{10} = -2A - 3B \quad \text{——equ * *}$$

Solving equ * and equ * * Simultaneously.

$$A + B = 0$$
$$-2A - 3B = -\frac{1}{10}$$

$$A = -B$$

$$-2(-B) - 3B = -\frac{1}{10}$$

$$-2(B) - 3B = -\frac{1}{10}$$

$$-B = -\frac{1}{10} \quad \therefore B = \frac{1}{10}.$$

$$A = -\frac{1}{10}.$$

$$x = -\frac{1}{10}e^{-2t} + \frac{1}{10}e^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t.$$

Write the steady-state solution of the system in the form of $X = k\sin(t+a)$

$x = -0.1e^{-2t} + 0.1e^{-3t} + 0.1\{\sin t + \cos t\}$

at steady state

i.e $t \Rightarrow \infty$

$\therefore 0.1e^{-3t} \to 0$ and $0.1e^{-3t} \to 0$ at steady state.

$x = 0.1\{\sin t + \cos t\} \Rightarrow$

$= 0.1\sin t + 0.1\cos t$

$x = k\sin(t+a)$

$\Rightarrow k\sin t\cos a + k\sin a\cos t$ ---(i)

Comparing (i) and (ii)

$k\sin a = 0.1$ ---(ii)

$k\cos a = 0.1$ ---(iii)

$k^2\sin^2 a + k^2\cos^2 a = 0.1^2 + 0.1^2$.

$k^2(\sin^2 a + \cos^2 a) = 0.02$.

$k^2 = 0.02$

$k = 0.1414$.

$\Rightarrow k\sin a = k\cos a = 0.1$

$\Rightarrow k\sin a = k\cos a$

$\sin a = \cos a$

$\frac{\sin a}{\cos a} = 1$

$\tan a = 1$

$a = \tan^{-1}(1)$

$a = 45^\circ$.

$\Rightarrow x = 0.1\{\sin t + \cos t\}$

$x = 0.1414\sin(t+45^\circ)$.