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CHEMICAL ENGINEERING
16/ENG01/009
EN9381

1) $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

Soln

$\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = 0$

Using auxiliary equation

$m^2 + 5m + 6 = 0$

$m_1 = -2, \quad m_2 = -3 \quad \therefore \quad x = Ae^{m_1 t} + Be^{m_2 t}$

$x = Ae^{-2t} + Be^{-3t}$

$P.I = f(t) = \cos t$

$x = A\cos t + B\sin t$

$\frac{dx}{dt} = -A\sin t + B\cos t$

$\frac{d^2x}{dt^2} = -A\cos t - B\sin t$

$\therefore \frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6 = \cos t$

$\Rightarrow (-A\cos t - B\sin t) + 5(-A\sin t + B\cos t) + 6(A\cos t + B\sin t) = \cos t$

$(-A\cos t - B\sin t) + (-5A\sin t + 5B\cos t) + (6A\cos t + 6B\sin t) = \cos t$

$(-A\cos t + 6A\cos t - B\sin t - 6B\sin t - 5A\sin t + 5B\cos t = \cos t$

collecting coefficient of like terms.

$5A + 5B = 1$

$-5A + 5B = 0$

Simultaneous elimination

$10B = 1$

$B = 1/10$

$\therefore 5A + 5B = 1$

$5A + 5/10 = 1$

$5A + 1/2 = 1$

$SA = 1 - 1/2$

$10A = 1$

$A = 1/10$

$P.I = \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$P.I = \frac{1}{10}(\cos t + \sin t)$

$x = C.F + P.I$

$x = \frac{1}{10}e^{-3t} + \frac{1}{10}e^{-2t} + \frac{1}{10}[\cos t + \sin t]$

2) Write a Matlab program to plot the relationship between x & t for $0 \leq t \leq 1$s using a step size of 0.01 unit.

Soln

```
-> Command window
-> clear
-> clc
-> close all
-> syms t
x = 0.1 * (exp(-3*t) - exp(-2*t) + cos(t) + sin(t))
tn = [0; 0.01 : 15]
xn = subs(x, tn)
figure(1)
plot(tn, xn)
~~frig~~ grid on
grid minor
axis tight
x label('time')
y label('vibrations')
```

2ii) Write a steady state solution of the system in form of $x = K\sin(t+a)$

Soln

$$x = \frac{1}{10}\left(e^{-3t} - e^{-2t} + \sin t + \cos t\right)$$

at steady state $\frac{dx}{dt} = 0$ i.e

change in x with time is zero (0)

$$\frac{dx}{dt} = \frac{1}{10}\left(-3e^{-3t} - e^{-2t} + \cos t - 3\sin t\right)$$

the exponentials result to zero (0)

$$0 = \cos t - \sin t$$

$$\cos t = \sin t$$

$$t = 45^\circ$$

$$x = \frac{1}{10}\left(\cos 45 + \sin 45\right)$$

from sinusoidal expression:

$A\cos wt + B\sin wt = K\cos(wt - \theta)$

But; $\cos(wt-\theta) = \sin(wt - \theta + 90^\circ)$

where $K = \sqrt{A^2+B^2} = \sqrt{(1/10)^2 + (1/10)^2} = \sqrt{\frac{1+1}{10^2}}$

$K = \sqrt{2}/10$

$\theta = 0^\circ$ (since w is in same phase)

Recall $x = K\sin(t+a)$

$$\frac{\sqrt{2}}{10} = \frac{\sqrt{2}}{10}\sin(45+a)$$

$$1 = \sin(45+a)$$

$$45 + a = \sin(\theta)$$

$$a = 90 - 45 = 45^\circ \text{ or } \pi/4$$

∴ the steady state solution is

$$x = \frac{\sqrt{2}}{10}\left(\sin 1 + \pi/4\right)$$