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Question 1:  $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

Solution

Using the auxiliary Equation

$$m^2 + 5m + 6 = 0$$

$$m^2 + 2m + 3m + 6 = 0$$

$$m(m+2) + 3(m+2) = 0$$

$$(m+3)(m+2) = 0$$

$$m_1 = -3, \quad m_2 = -2$$

$$\therefore \text{C.F.} = x = A_1 e^{-3t} + B_1 e^{-2t}$$

$$\text{P.I.} = x = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

$$\therefore (-C \cos t - D \sin t) + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) - (\cos t - C \cos t) - (D \sin t - 5C \cos t) + (5D \cos t + 6C \cos t) + (5D \sin t - \cos t)$$

$$(-C + 5D + 6C) \cos t + (-D - 5C + 6D) \sin t = \cos t$$

$$5D + 5C = 1$$

$$-5C + 5D + 5C + 5D = 0$$

$$5D = 5C$$

$$D = C$$

$$+ 5(C) + 5C = 1$$

$$5C + 5C = 1$$

$$10C = 1$$

$$C = \frac{1}{10}$$

$$\therefore C = D = \frac{1}{10}$$

$$PI = \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$\text{General Solution } \rightarrow X = AC + Be + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$X = Ae^{-3t} + Be^{2t} + \frac{1}{10}(\cos t + \sin t)$$

When  $t=0$ ,  $X=0.1$  and  $dX/dt=0$

$$0.1 = Ae^0 + Be^0 + \frac{1}{10}(\cos 0 + \sin 0)$$

$$0.1 = A + B + \frac{1}{10}(1+0)$$

$$A + B = 0$$

$$A = -B$$

$$Xe = -3Ae^{3t} + 2Be^{2t} + \frac{1}{10}(\cos t + \sin t)$$

$$dXe = 0, \quad 1=0$$

$$0 = +3Ae^0 - 2Be^0 + \frac{1}{10}(-\sin 0 + \cos 0)$$

$$0 = -3Ae^0 - 2B + \frac{1}{10}(0+1)$$

$$3A + 2B = 0.1$$

$$3A + 2B = \frac{1}{10}$$

$$3A + 2B = \frac{1}{10}$$

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From equation \*  $A = -B$

$$3(-B) + 2B = 0.1 \Rightarrow -B = 0.1$$

$$-3B + 2B = 0.1 \Rightarrow -B = 0.1$$

$$-B = 0.1 \Rightarrow B = -0.1$$

$$\therefore x = \frac{1}{10} (e^{-3t} - e^{-2t} + \cos t + \sin t)$$

Question 2:  $x = \frac{1}{10} (-e^{-2t} + e^{-3t})$

Solution.

Using steady state solution

$$x = \frac{1}{10} (-e^{-3t} + e^{-2t}) + (\cos t + \sin t)$$

The point of transient of the equation will not be considered since we are dealing with a steady state.

Since  $\frac{dx}{dt} = 0$  for a steady state

$$\frac{dx}{dt} - \frac{1}{10} (-\sin t + \cos t) = 0$$

$$-\sin t + \cos t = 0$$

$$\cos t = \sin t$$

$$\therefore t = 45^\circ$$

$$x = \frac{1}{10} (\cos 45 + \sin 45) = \frac{\sqrt{2}}{10}$$

From the sinusoidal expression

$$A \cos \omega t + B \sin \omega t + K \cos(\omega t - \theta)$$

$$\text{but } \cos(\omega t - \theta) = \sin(\omega t - \theta) + 70^\circ$$

where

$$K = \sqrt{A^2 + B^2} = \sqrt{(10)^2 + (10)^2}$$

$$= \sqrt{10^2 + 10^2}$$

$$K = \frac{\sqrt{2}}{10}$$

$$\theta = 0^\circ$$

$$\text{Recall } x = K \sin(\omega t + \phi)$$

$$\frac{x}{\sqrt{2}}$$

$$K = \sqrt{2}/10 \sin 45^\circ + \tau$$

$$1 = \sin(45^\circ + \tau)$$

$$45^\circ + \tau = \sin^{-1}(1)$$

$$\alpha = 90^\circ - 45^\circ = 45^\circ = \pi/4$$

The steady state solution is,

$$x = \frac{\sqrt{2}}{10} \sin(\omega t + \pi/4)$$