

07/10/2018

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Matric NO: 17/ENG 04/085

Course Code: ENG 381

$$\frac{d^2 x}{dt^2} + 5 \frac{dx}{dt} + 6x = \cos t \dots (i)$$

when  $t=0$   $x=0.1$  and  $\frac{dx}{dt}=0$ 

Using auxiliary equation method

$$\frac{d^2 x}{dt^2} + 5 \frac{dx}{dt} + 6x = 0$$

$$\text{Let } x = Ae^{kt}$$

$$\frac{dx}{dt} = kAe^{kt} \quad \frac{dx}{dt} = kx$$

$$\frac{d^2 x}{dt^2} = k^2 Ae^{kt} \quad \frac{d^2 x}{dt^2} = k^2 x$$

$$\therefore k^2 x + 5kx + 6x = 0$$

divide through by  $x$ 

$$\frac{k^2 x}{x} + \frac{5kx}{x} + \frac{6x}{x} = 0$$

$$k^2 + 5k + 6 = 0 \rightarrow \text{auxiliary equation}$$

$$k(k+2) + 3(k+2) = 0$$

$$(k+3)(k+2) = 0$$

$$\therefore k_1 = -3 \text{ or } k_2 = -2$$

Hence;

$$\text{Since } y = Ae^{k_1 x} + Be^{k_2 x}$$

$$x = Ae^{-2t} + Be^{-3t} \rightarrow \text{complementary function}$$

for  $\cos t$ 

$$x = A \cos t + B \sin t$$

$$\frac{dx}{dt} = -A \sin t + B \cos t$$

$$\frac{d^2 x}{dt^2} = -A \cos t - B \sin t$$

from equation (i)

$$\frac{d^2 x}{dt^2} + 5 \frac{dx}{dt} + 6x = \cos t$$



Paper

Substituting into equation (1)  
 $-A \cos t - B \sin t + 5(-A \sin t + B \cos t) + 6(A \cos t + B \sin t) = \cos t$   
 $= (\cos t (-A + 5B + 6A) + \sin t (-B - 5A + 6B)) \cos t$

$$-A + 5B + 6A = 1 \cos t$$

$$-A + 5B + 6A = 1$$

$$-5A + 6B - B = 0$$

$$-A + 5A + 5B = 1 \quad \dots (2)$$

$$-5A + 5B = 0 \quad \dots (3)$$

$$10A = 1$$

$$A = \frac{1}{10}$$

Substituting A into equation (2)

$$5\left(\frac{1}{10}\right) + 5B = 1$$

$$\frac{1}{2} + 5B = 1$$

$$5B = 1 - \frac{1}{2}$$

$$B = \frac{1}{2} \div 5$$

$$B = \frac{1}{10}$$

$\therefore x = \frac{1}{10} \cos t + \frac{1}{10} \sin t \rightarrow$  Partial Integral

General solution = complementary function + partial integral

$$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - \frac{\sin t}{10} + \frac{\cos t}{10}$$

$$\text{where } x=0.1 \quad \frac{dx}{dt} = 0 \quad t=0$$

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10} \cos(0) + \frac{1}{10} \sin(0)$$

$$0.1 = A + B + \frac{1}{10} \cdot 10$$

$$0.1 - \frac{1}{10} = A + B = 0 \quad A = -B$$

Then;

$$0 = 2Ae^{-2(0)} - 3Be^{-3(0)} - \frac{1}{10} \sin(0) + \frac{1}{10} \cos(0)$$

$$0 = -2A - 3B - \frac{1}{10}$$

$$-2A - 3B = -\frac{1}{10}$$

$$-2(-B) - 3B = -\frac{1}{10}$$

$$2B - 3B = -\frac{1}{10}$$

$$-B = -\frac{1}{10}$$

$$B = \frac{1}{10}$$



$$x = -\frac{1}{10}e^{-2t} + \frac{1}{10}e^{-2t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$$

$$x = \frac{1}{10}(-e^{-2t} + e^{-2t} + \cos t + \sin t)$$

From the graph: amplitude  $k = 0.41$

$$\text{Period (T)} = 14.9 - 8.7 = 6.2 \text{ s}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{6.2} = 0.1 \text{ rad/s or } \frac{\pi}{6} \text{ rad}$$

$a$  = the value of  $t$  for which  $\sin t = \cos t$   
 $a = 45^\circ$  or  $\frac{\pi}{4} \text{ rad}$

In the form  $k \sin(t+a)$ , the steady state solution is given as

$$0.41 \sin(t + 45^\circ)$$