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Solution to Assignment

i $\frac{d^2x}{dx^2} + 5\frac{dx}{dt} + 6x = \cos t$ ——— *

$m^2 + 5m + 6 = 0$

$a = 1 \quad b = 5 \quad c = 6$ using

$m = \frac{-5 \pm \sqrt{5^2 - 4(1)(6)}}{2(1)}$

$= \frac{-5 \pm \sqrt{25 - 24}}{2}$

$= \frac{-5 + \sqrt{1}}{2} \quad = \frac{-5 + 1}{2}$

$= \frac{-5 + 1}{2} \quad \text{or} \quad \frac{-5 - 1}{2}$

$= \frac{-4}{2} \quad \text{or} \quad \frac{-6}{2}$

$m = -2 \text{ or } -3$

$\therefore x = Ae^{-2t} + Be^{-3t}$

$x = C\cos t + D\sin t$

$\frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

Sub x, dx/dt and $\frac{d^2x}{dt^2}$ into eqn *

$(-C\cos t - D\sin t) + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

$\cos t(-C + 5D + 6C) + \sin t(-D - 5C + 6D) = \cos t$

Equating the co-efficients

$5D + 5C = 1$ ——— (1)

$-5D - 5C = 0$ ——— (2)

$10C = 1$

$c = 1/10$

$c = 0.1$

Sub c into eqn 1

$5D + 5(\frac{1}{10^2}) = 1$

$5D + \frac{1}{2} = 1$

$5D = 1 - 1/2$

$5D = 1/2$

$D = 1/10 = 0.1$

$x = Ae^{-2t} + Be^{-3t} + 0.1\cos t + 0.1\sin t$

when $t = 0$, $x = 0.1$, $dx/dt = 0$

$0.1 = Ae^{-2(0)} + Be^{-3(0)} + 0.1\cos 0 + 0.1\sin 0$

$0.1 = Ae^0 + Be^0 + 0.1 + 0$

$0.1 = A + B + 0.1$

$A + B = 0$ ——— (3)

$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - 0.1\sin t + 0.1\cos t$

$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} - 0.1\sin 0 + 0.1\cos 0$

$0 = -2A - 3B - 0 + 0.1$

$2A + 3B = 0.1$ ——— (4)

$A + B = 0$ $\times 2$

$2A + 3B = 0.1$

$2A + 2B = 0$

$-\ 2A + 3B = 0.1$

$-B = 0.1$

$B = -0.1$

sub B into eqn 3

$A - 0.1 = 0$

$A = 0.1$

$\therefore x = 0.1e^{-2t} - 0.1e^{-3t} + 0.1\cos t + 0.1\sin t$

iii from the graph,

Amplitude $K = 0.141$; $A\sin(\omega t + \phi)$

Period $T = 14.9 - 8.7$

$T = 6.2\ sec$

$\therefore \omega = \frac{2\pi}{T} = \frac{2\pi}{6.2} = 1.0\ rad/sec$ or $\pi/3\ rad$

q = the value of t for which $\sin t = \cos t$

$\therefore q = 45^\circ$ or $\pi/4\ rad$

Hence, in the form $K\sin(t+a)$, the steady state solution is given as

$0.141 \sin(t + 45^\circ)$

or

$$\frac{\sqrt{2}}{10} \sin\left(t + \frac{\pi}{4}\right)$$