6. $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

Auxiliary equation

$m^2 + 6m + 6 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{-5 \pm \sqrt{25 - 24}}{2}$

$m = \frac{-5 \pm 1}{2}$

$m = -2$ or 3

C.f $\rightarrow$ $x = Ae^{-2t} + Be^{-3t}$

P.I

$x = C\cos t + D\sin t$

$\frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

$(-C + 5D + 6C)\cos t + (-D - 5C + 6D)\sin t = \cos t$

Equating

$-C + 5D + 6C = 1$

$5C + 5D = 1$ --①

$-D - 5C + 6D = 0$

$-5C + 5D = 0$ ----②

$5C + 5D = 1$

$-5C + 5D = 0$

$10C = 1$

$C = \frac{1}{10}$

$5(t) + 5D = 1$

$\frac{1}{2} + 5D = 1$

$5D = 1 - \frac{1}{2}$

$5D = \frac{1}{2}$

$D = \frac{1}{10}$

$x = \frac{1}{10}\cos t + \frac{1}{10}\sin t$

Generic Solution $= CF + PI$

$= Ae^{-2t} + Be^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

(11) Write a matlab program to plot the relationship between x if for $0 \leq t \leq t_1$ Unit Using a Step Size of 0.01 Unit.

Solution

- Command window
- Clear
- clc
- Close all
- Syms t

```
x = 0.1*(exp(-3*t) - exp(-2*t)) + (cos(t) + sin(t))
tn = [0, 0.01 : 15]
xn = subs(x, tn)
figure(1)
plot(tn, xn)
grid on
grid minor
axis tight
```

x label ('time')
y label ('vibrations')

(3) Write the steady state solution of the System in form of $x = k\sin(t + a)$

Solution

$$x = \frac{1}{10}(e^{-3t} - e^{-2t} + \sin t + \cos t)$$

At steady state $\frac{dx}{dt} = 0$ i.e

Change in x with time is zero

$$\therefore \frac{dx}{dt} = \frac{1}{10}(-3e^{-3t} - e^{-2t} + \cos t - 3\sin t)$$

nb: the exponentials result zero

$$0 = \cos t - \sin t$$

$$\cos t = \sin t$$

$$t = 45^\circ$$

$$x = \frac{1}{10}(\cos 45 + \sin 45) = \frac{\sqrt{2}}{10}$$

from Sinosodial expression

$$A\cos wt + P\sin wt = k\cos(wt - \theta)$$

But; $\cos(wt - \theta) = \sin(wt - \theta + 90^\circ)$

where; $k = \sqrt{A^2 + B^2} = \sqrt{(1/10)^2 + (1/10)^2} = \sqrt{\frac{1+1}{10^2}}$

$$k = \frac{\sqrt{2}}{10}$$

$\theta = 0^\circ$ (since its in same phase)

Recall $x = k\sin(t + a)$

$$\frac{\sqrt{2}}{10} = \frac{\sqrt{2}}{10}\sin(45 + a)$$

$\varphi = 90 - 45 = 45^\circ \leftarrow \pi/4$

The steady state solution is;

$$x = \frac{\sqrt{2}}{10} (\sin t + \pi/4)$$