SOLUTION:

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Auxiliary eqn $= m^2 + 5m + 6 = 0$

$$m^2 + 3m + 2m + 6 = 0$$

$$m(m+3) + 2(m+3) = 0$$

$$(m+2)(m+3) = 0$$

$$m_1 = -2 \text{ and } m_2 = -3$$

Complimentary function;

$$C.F = Ae^{m_1 t} + Be^{m_2 t}$$

$$C.F = Ae^{-2t} + Be^{-3t}$$

P.I $x = C\cos t + D\sin t$; $dx/dt = -C\sin t + D\cos t$

$$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$$

Putting d^2x/dt^2 and dx/dt in orginal equation

$$\rightarrow -C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$$

$$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$$

$$-\cos t + 6C\cos t + 5D\cos t - D\sin t - 5C\sin t + 6D\sin t = \cos t$$

$$5C\cos t + 5D\cos t + 5D\sin t - 5C\sin t = \cos t$$

$$5C + 5D = 1 \quad \text{----} (1)$$

$$5D - 5C = 0 \quad \text{----} (2)$$

from (2)

$$5D = 5C$$

$$D = C$$

Put it in eq 1

$$5C + 5C = 1$$

$$10C = 1 \quad \therefore C = 1/10$$

Since D = C

therefore D = 1/10

$P.I = \frac{1}{10}\cos t + \frac{1}{10}\sin t$

General Solution = C.F + P.I

$x = Ae^{-2t} + Be^{-3t} + 1/10\cos t + 1/10\sin t$

when $t = 0$, $x = 0.1$ and $\frac{dx}{dt} = 0$

$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} + \frac{1}{10}\sin t + \frac{1}{10}\cos t$

$0.1 = Ae^{-2t} + Be^{-3t} + 1/10\cos t + 1/10\sin t$

$0.1 = A + B + 1/10 + 0$

$0.1 = A + B + 1/10$

$A + B = 0.1 - 1/10$

$A + B = 0 \quad \therefore A = -B$

$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} - 1/10\sin(0) + 1/10\cos(0)$

$0 = -2A - 3B - 0 + 1/10$

$-1/10 = -2A - 3B$; $-1/10 = -2(-B) - 3B$

$-1/10 = -B$

$B = 1/10$

$A = -B = -1/10$

$\therefore x = -\frac{1}{10}e^{-2t} + 1/10\,e^{-3t} + 1/10\cos t + 1/10\sin t$

3) $x = A\sin(ct + a)$

$x = -0.1e^{-2t} + 0.1e^{-3t} + 0.1\sin t + 0.1\cos t$

(a) steady state

$t \to \infty$

$\therefore 0.1e^{-3t} \to 0$ & $0.1e^{-2t} = 0$

$x = 0.1 [\sin t + \cos t]$

$x = k \sin(t + \alpha)$

$\Rightarrow k \sin t \cos\alpha + k \sin\alpha \cos t$ from trigonometry

$k \sin t \cos\alpha + k \sin\alpha \cos t = 0.1 \sin t + 0.1 \cos t$

$k \cos\alpha = 0.1 \qquad k \sin\alpha = 0.1$

$k^2 \sin^2\alpha + k^2 \cos^2\alpha = 0.1^2 + 0.1^2$

$k^2 (\sin^2\alpha + \cos^2\alpha) = 0.02$

$\sin^2\alpha + \cos^2\alpha = 1$

The equation becomes

$k^2 = 0.02$

$k = 0.1414.$

and $k \sin\alpha = k \cos\alpha$

$\sin\alpha = \cos\alpha$

$\frac{\sin\alpha}{\cos\alpha} = 1$

$\frac{\sin}{\cos} = \tan$

$\tan\alpha = 1$

$\alpha = \tan^{-1} 1$

$\alpha = 45^\circ$

$x = 0.1 (\cos t + \sin t) = k \sin(t + \alpha) = 0.1414 \sin(t + 45^\circ)$

$x = 0.1414 \sin(t + 45^\circ)$