

Assignment

$$\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t$$

$$\text{let } x = e^{kt}$$

$$\frac{dx}{dt} = ke^{kt} = kx \quad \frac{d^2x}{dt^2} = k^2e^{kt} = k^2x$$

$$k^2x + 5kx + 6x = 0$$

$$x(k^2 + 5k + 6) = 0$$

∴ divide all through by x

$$k^2 + 5k + 6 = 0 \quad \text{Auxiliary Equation}$$

$$k = \frac{-5 \pm \sqrt{5^2 - 4 \times 1 \times 6}}{2 \times 1}$$

$$k_1 = \frac{-5 + 1}{2}$$

$$k_2 = \frac{-5 - 1}{2}$$

$$k_1 = -2$$

$$k_2 = -3$$

$$x = Ae^{-2t} + Be^{-3t} \quad \text{C.F.}$$

General Form

Particular

$$x = (\cos t + 1) \sin t$$

$$\frac{dx}{dt} = -(\sin t + 1) \cos t$$

$$\frac{d^2x}{dt^2} = -\cos t - \sin t$$

$$[-(\cos t - \sin t) + 5(-\cos t + \sin t) + 6(\cos t + \sin t)]$$

$$[-(\cos t - \sin t) - 5(\sin t + \cos t) + 6(\cos t + \sin t) = \cos t$$

$$(-1 + 5 + 6) \cos t + (1 - 5 + 6) \sin t$$

$$(5 + 5) \cos t - (5 - 5) \sin t = \cos t$$

$$5 + 5 = 1$$

$$-5 + 5 = 6$$

$$10 \cos t = 1$$

$$C = \frac{1}{10}$$

$$S(\frac{1}{10}) + 5D = 1$$

$$\frac{1}{2} + 5D = 1$$

$$\frac{1}{2} + 5D = 1$$

$$5D = 1 - \frac{1}{2}$$

$$5D = \frac{1}{2}$$

$$D = \frac{1}{2} \times \frac{1}{5}$$

$$D = \frac{1}{10}$$

Complete General Solution, where $t = 0$, $x = 0.1$, $\frac{dx}{dt} = 0$

$$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10} \cos t = \frac{1}{10} \sin t$$

$$8 Ae^{-2t} + 3Be^{-3t} + \frac{1}{10} \cos t = \frac{1}{10} \sin t$$

~~$$Ae^{-2t} + Be^{-3t} + \frac{1}{10} \cos t = \frac{1}{10} \sin t$$~~

When $t = 0$

$$\frac{dx}{dt} = -2Ae^{-2t} + 3Be^{-3t} - \frac{1}{10} \sin t + \frac{1}{10} \cos t$$

When $t = 0$, $x = 0.1$

$$0.1 = A + B + \frac{1}{10}$$

$$0.1 = A + B + 0.1$$

$$0.1 - 0.1 = A + B$$

$$A + B = 0 \quad \text{--- (1)}$$

$$0 = -2Ae^{-2t} - 3Be^{-3t} - \frac{1}{10} \sin t + \frac{1}{10} \cos t$$

When $t = 0$

$$0 = -2A - 3B + \frac{1}{10}$$

$$-\frac{1}{10} = -2A - 3B$$

$$-0.1 = -2A - 3B \quad \text{--- (2)}$$

$$0 = A + B \quad \text{--- (1)}$$

$$-0.1 = -2A - 3B$$

$$0 = 2A + 2B$$

$$-0.1 = 0.1B$$

$$0.1 = -B$$

$$B = -0.1$$

Sub in eqn (1)

$$0 = A + B \quad \text{--- (1)}$$

$$0 = A - 0.1$$

$$A = 0.1$$

$$x = 0.1e^{-2t} - 0.1e^{-3t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$