Time integration

## Assignment

1. $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$ ...... ①

i Auxillary equation:

$m^2 + 5m + 6x = 0$

Using quadratic equation

$a = 1$, $b = 5$, $c = 6$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-5 \pm \sqrt{5^2 - 4 \times 1 \times 6}}{2 \times 1}$$

$$m = \frac{-5 \pm \sqrt{25 - 24}}{2}$$

$$m_1 = \frac{-5 + 1}{2}, \quad m_2 = \frac{-5 - 1}{2}$$

$$m_1 = -4/2 = -2 \quad m_2 = -6/2 = -3$$

$m_1 = -2$, $m_2 = -3$

$$x = Ae^{-2t} + Be^{-3t}$$

Particular Integral $= A\cos t + B\sin t = x$

$$\frac{dx}{dt} = -A\sin t + B\cos t$$

$$\frac{d^2x}{dt^2} = -A\cos t - B\sin t$$

Substituting in equ (1)

$$-A\cos t - B\sin t + 5\left[-A\sin t + B\cos t\right] + 6\left[A\cos t + B\sin t\right] =$$

$$6A\cos t + 6B\sin t = \cos t$$

$m_1 = -4/2 = -2 \quad m_2 = -6/2 = -3$

$m_1 = -2, \; m_2 = -3$

$x = Ae^{-2t} + Be^{-3t}$

Particular Integral $= A\cos t + B\sin t = x$

$\frac{dx}{dt} = -A\sin t + B\cos t$

$\frac{d^2x}{dt^2} = -A\cos t - B\sin t$

Substituting in equ (1)

$-A\cos t - B\sin t + 5[-A\sin t + B\cos t] + 6[A\cos t + B\sin t] = \cos t$

$-A\cos t - B\sin t - 5A\sin t + 5B\cos t + 6A\cos t + 6B\sin t = \cos t$

$-A\cos t + 5B\cos t + 6A\cos t - B\sin t - 5A\sin t + 6B\sin t = \cos t$

$\cos t(-A + 5B + 6A) + \sin t(-B - 5A + 6B) = \cos t$

$-A + 5B + 6A = 1 \quad \ldots \text{(ii)} \quad ; \quad 5A + 5B = 1$

$-B - 5A + 6B = 0 \quad \ldots \text{(iii)} \quad ; \quad 5B - 5A = 0$

$5A + 5B = 1$

$-5A + 5B = 0$

$\cancel{5A} \cancel{-} \cancel{-5B} \cancel{+1} \quad \cancel{5}B = \cancel{5}A$

$B = A$

$5A + 5A = 1$

$10A = 1$

$A = 1/10$

$B = 0/10$

General Particular integer $= \frac{\cos t}{10} + \frac{\sin t}{10}$

General solution = Particular integer + Complementary function

$x$ = general solution $= \frac{\cos t}{10} + \frac{\sin t}{10} + Ae^{-2t} + Be^{-3t}$

when $t = 0$, $x = 0.1$ $\frac{dx}{dt} = 0$

$\frac{dx}{dt} = -\frac{\sin t}{10} + \frac{\cos t}{10} - 2Ae^{-2t} - 3Be^{-3t}$

$0.1 = \frac{\cos(0)}{10} + \frac{\sin(0)}{10} + Ae^{-2(0)} + Be^{-3(0)}$

$0.1 = \frac{1}{10} + A + B$

$\frac{1}{10} = \frac{1}{10} + A + B$

$A + B = \frac{1}{10} - \frac{1}{10}$

$A + B = 0$

when $\frac{dx}{dt} = 0$

$0 = -\frac{\sin(0)}{10} + \frac{\cos(0)}{10} - 2Ae^{-2(0)} - 3Be^{-3(0)}$

when $t = 0$, $x = \ldots \frac{}{dt}$

$$\frac{dx}{dt} = -\frac{\sin t}{10} + \frac{\cos t}{10} - 2Ae^{-2t} - 3Be^{-3t}$$

$$0.1 = \frac{\cos(0)}{10} + \frac{\sin(0)}{10} + Ae^{-2(0)} + Be^{-3(0)}$$

$$0.1 = \frac{1}{10} + A + B$$

$$\frac{1}{10} = \frac{1}{10} + A + B$$

$$A + B = \frac{1}{10} - \frac{1}{10}$$

$$A + B = 0$$

when $\frac{dx}{dt} = 0$

$$0 = -\frac{\sin(0)}{10} + \frac{\cos(0)}{10} - 2Ae^{-2(0)} - 3Be^{-3(0)}$$

$$0 = 0 + \frac{1}{10} - 2A - 3B$$

$$-\frac{1}{10} = -(2A + 3B)$$

$$\frac{1}{10} = 2A + 3B$$

$A + B = 0$ ...... (iv)

$2A + 3B = 0.1$ .. (v)

$A = -B$

$2(-B) + 3B = 0.1$

$-2B + 3B = 0.1$

$B = 1/10 \; ; \; A = -1/10$

$x$ = general solution = $\dfrac{\cos t}{10} + \dfrac{\sin t}{10} + \dfrac{e^{-2t}}{10} + \dfrac{e^{-3t}}{10}$

3- $x = K\sin(t+a)$

$x = -0.1e^{-2t} + 0.1e^{-3t} + 0.1\sin t + 0.1\cos t$

steady state

$t \to \infty$

$\therefore 0.1e^{-3t} \Rightarrow 0 \quad ; 0.1e^{-2t} = 0$

$x = 0.1(\sin t + \cos t)$

$x = K\sin(t+a)$

$= K\sin t\cos a + K\sin a\cos t = 0.1\sin t + 0.1\cos t$

~~$K^2 \sin t\cos a + K\sin a$~~

$K^2\sin^2 a + K^2\cos^2 a = 0.1^2 + 0.1^2$

$K^2(\sin^2 a + \cos^2 a) = 0.02$

$\sin^2 a + \cos^2 a = 1$

$K^2 = 0.02$

$K = 0.1414$

$K\sin a = K\cos a$

$\sin a = \cos a$

$\frac{\sin a}{\cos a} = 1 = \tan a$

$a = \tan^{-1}(1)$ since $1 = \tan a$

$a = 45^\circ$

$x = 0.1(\cos t + \sin t) = K\sin(t+a) = 0.1414\sin(t+45)$

$x = 0.1414\sin(t+45)$