Question 1: $\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t$

Solution

Using the auxilliary Equation

$m^2 + 5m + 6 = 0$

$m^2 + 2m + 3m + 6 = 0$

$m(m+2) + 3(m+2) = 0$

$(m+3)(m+2)$

$m_1 = 3, \quad m_2 = -2$

$\therefore CF = x = Ae^{-3t} + Be^{-2t}$

$P.I = x = C\cos t + D\sin t$

$\frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

$\therefore (-C\cos t - D\sin t) + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) - (\cos t - C\cos t) - (D\sin t - 5C\cos t) + (5D\cos t + 6C\cos t) + (6D\sin t - \cos t).$

$(-C + 5D + 6C)\cos t + (-D - 5C + 6D)\sin t = \cos t$

$5D + 5C = 1$

$\cancel{-5C+}\ -5C + 5D = 0$

$5D = 5C$

$D = C$

$+5(C) + 5C = 1$

$5C + 5C = 1$

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$10C = 1$

$C = \frac{1}{10}$

$\therefore C = D = \frac{1}{10}$

$PI = \frac{1}{10} \cos t + \frac{1}{10} \sin t$

General Solution $\rightarrow x = Ae^{-3t} + Be^{-2t} + \frac{\cos t}{10} + \frac{\sin t}{10}$

$x = Ae^{-3t} + Be^{-2t} + \frac{1}{10}(\cos t + \sin t)$

When $t = 0$, $x = 0.1$ and $dx/dt = 0$

$0.1 = Ae^{0} + Be^{0} + \frac{1}{10}(\cos 0 + \sin 0)$

$0.1 = A + B + \frac{1}{10}(1 + 0)$

$A + B = 0$

$A = -B$ .... eqn $*$

$\frac{dx}{dt} = -3Ae^{-3t} + 2Be^{-2t} + \frac{1}{10}(-\sin t + \cos t)$

$\frac{dx}{dx} = 0, \quad t = 0$

$0 = -3Ae^{0} - 2Be^{0} + \frac{1}{10}(-\sin 0 + \cos 0)$

$0 = -3Ae^{0} - 2B + \frac{1}{10}(0 + 1)$

$3A + 2B = 0.1$

$3A + 2B = \frac{1}{10}$

From equation $*$ $\quad A = -B$

$3(-B) + 2B = 0.1$

$-3B + 2B = 0.1$

$-B = 0.1$

$\therefore x = \frac{1}{10}\left(e^{-3t} - e^{-2t} + \cos t + \sin t\right)$

Question 2: $x = \frac{r}{10}\left(-e^{-2t} + e^{-3t}\right) + (\cos t + \sin t)$

Solution.

Using steady state Solution

$x = \frac{r}{10}\left(-e^{-2t} + e^{-3t}\right) + (\cos t + \sin t)$

The point of transcent of the equation will not be considered Since we are dealing with a steady state.

Since $\frac{dx}{dt} = 0$ for a steady state

$\frac{dx}{dt} - \frac{1}{10}(-\sin t + \cos t) = 0$

$-\sin t + \cos t = 0$

$\cos t = \sin t$

$\therefore t = 45°$

$x = \frac{1}{10}(\cos 45 + \sin 45) = \frac{\sqrt{2}}{10}$

From the Sinosodial expression

$A\cos wt + B\sin wt + K\cos(wt - \theta)$

but $\cos(wt - \theta) = \sin(wt - \theta) + 90°)$

Where

$K = \sqrt{A^2 + B^2} = \sqrt{(1/10)^2 + (1/10)^2}$

$= \sqrt{11/102}$

$K = \frac{\sqrt{2}}{10}$

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$\theta = 0^\circ$

Recall $\frac{x}{\sqrt{2}} = K\sin(1 + a)$

$K = \sqrt{2}/10 \sin 45 + a$

$1 = \sin(45 + a)$

$45 + a = \sin^{-1}(1)$

$a = 90 - 45 = 45^\circ = \pi/4$

The steady state solution,

$x = \frac{\sqrt{2}}{10} \sin(t + \pi/4)$