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Assignment

The dynamic model of a body in motion performing damped force Vibrations is as in Equation (1)

$$\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t \quad (1)$$

Given that when $t=0$, $x=0.1$ and $\frac{dx}{dt}=0$

(i) using the auxillary Equation method, obtain the solution of the model in form of an Expression having x as a function of t,

(ii) Write a MATLAB Program to plot the relationship b/w x and t for $0 \le t \le 15$ Unit using a step size of 0.01unit, and

(iii) Write the steady-state solution of the system in form of $x = K\sin(Gt + a)$

Solution

(i) $\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t$

let $x = Ae^{Kt}$

$x' = \frac{dx}{dt} = KAe^{Kt} = Kx$

$x'' = \frac{d^2x}{dt^2} = K^2Ae^{Kt} = K^2x$

let R.H.S = 0

$K^2x + 5Kx + 6x = 0$

$K^2 + 5K + 6 = 0$ (by using factorization method)

$K^2 + 3K + 2K + 6 = 0$

$K(K+3) + 2(K+3) = 0$

$(K+2)(K+3) = 0$

$K_1 = -2$ \& $K_2 = -3$

for Real \& distinct roots

$x = Ae^{k_1 t} + Be^{k_2 t}$

$x = Ae^{-2t} + Be^{-3t}$ {Complementary function}

from R.H.S

$x = \cos t$

General form of the function, $x = C\cos t + D\sin t$

$$x' = \frac{dx}{dt} = -C\sin t + D\cos t$$

$$x'' = \frac{d^2x}{dt^2} = -C\cos t - D\sin t = -(C\cos t + D\sin t)$$

$$-(C\cos t + D\sin t) + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$$

$$\cos t(-C + 5D + 6C) + \sin t(-D - 5C + 6D) = \cos t$$

$$\cancel{\cos t}(-C + 5D + 6C) = \cancel{\cos t}$$

$$\sin t(-D - 5C + 6D) = 0$$

$$-C + 5D + 6C = 1 \qquad 5D + 5C = 1 \quad \text{---} (1)$$

$$-D - 5C + 6D = 0 \qquad 5D - 5C = 0 \quad \text{---} (2)$$

equ (1) - (2) $\quad 10C = 1$

$$C = 1/10$$

~~sub C = 1/10 into equ (2)~~

$$5D - 5(1/10) = 0$$

$$5D = 1/5$$

$$D = 1/10$$

$$x = \frac{1}{10}\cos t + \frac{1}{10}\sin t \quad \text{\{particular Integral\}}$$

Complete General solution [CGS] = Complementary function (C.F) + particular Integral (P.I)

$$x = Ae^{-2t} + Be^{-3t} + 1/10(\cos t + \sin t)$$

When $t = 0$, $x = 0.1$ and $x' = 0$

$$x' = dx/dt = -2Ae^{-2t} - 3Be^{-3t} - 1/10\sin t + 1/10\cos t$$

$$x' = -(2Ae^{-2t} + 3Be^{-3t} + 1/10\sin t - 1/10\cos t)$$

$0.1 = A + B + 0.1$ $\qquad A + B = 0$ —— ① × ②

$0 = -(2A + 3B - 0.1)$ $\qquad -2A - 3B = -0.1$ —— ②

$\qquad 2A + 2B = 0$ —— ③

equ ② + ③ $\quad -B = -0.1$

$B = 0.1$

sub $B = 0.1$ into equ ①

$A + 0.1 = 0$

$A = -0.1$

Complete particular solution {C.P.S}

$$x = -\tfrac{1}{10}e^{-2t} + \tfrac{1}{10}e^{-3t} + \tfrac{1}{10}(\cos t + \sin t)$$

$$x = -0.1(e^{-2t} - e^{-3t} - \cos t - \sin t)$$

② solution

- Command Window
- clear
- clc
- close all
- syms

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x = 0.1 x (exp(-3*t) - exp(-2*t) + (cost) + (sint))
tn = [0:0.01:15]
tn = subs(x,tn)
figure 1.
Plot (tn, xn)
Axis tight
x label ('time')
y label ('Vibration')
```

③ solution

$$x = \tfrac{1}{10}(e^{-3t} - e^{-2t} + \sin t + \cos t)$$

at steady state $\frac{dx}{dt} = 0$ 1.8

$$\frac{dx}{dt} = \frac{1}{10}\left(-3e^{-3t} - e^{-2t} + \cos t - \sin t\right)$$

(no more exponential decay)

∴ the exponential result zero because it is at steady state

$0 = \cos t - \sin t$

$\cos t = \sin t$

$t = 45^0$

$$x = \frac{1}{10}(\cos 45 + \sin 45) = \frac{\sqrt{2}}{10}$$

$A\cos wt + B\sin wt = K\cos(wt - \theta)$

But ; $\cos(wt - \theta) = \sin(wt - \theta + 90^0)$

Where, $K = \sqrt{A^2 + B^2}$

$= \sqrt{\frac{1}{10}^2 + \frac{1}{10}^2} = \frac{\sqrt{2}}{10}$

$\theta = 0^0$

Recall $x = K\sin(t + a)$

$$\frac{\sqrt{2}}{10} = \frac{\sqrt{2}}{10}\sin(45 + a)$$

$a = 90 - 45 = 45^0$

Steady state equation

$$x = \frac{\sqrt{2}}{10}\left(\sin 1 + \frac{\pi}{4}\right)$$