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16/ENG05/004
MECHATRONICS

ENG 381 · Assignment 1

(i)

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Obtaining C.F

assuming $f(t) = 0$

$$m^2 + 5m + 6 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-5 \pm \sqrt{5^2 - 4(1 \times 6)}}{2} = \frac{-5 \pm \sqrt{25 - 24}}{2} = \frac{-5 \pm \sqrt{1}}{2}$$

$$m = \frac{-5 \pm 1}{2} = \frac{-4}{2} \text{ or } \frac{-6}{2} = -2 \text{ or } -3$$

$$C.F \quad x = Ae^{-2t} + Be^{-3t}$$

Obtaining P.I

$$\text{If } f(t) = \cos t$$

$$x = C\cos t + D\sin t$$

$$\frac{dx}{dt} = -C\sin t + D\cos t$$

$$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$$

Substituting back into the equation

$$\cos t = \left(-C\cos t - D\sin t\right) + 5\left(-C\sin t + D\cos t\right) + 6\left(C\cos t + D\sin t\right)$$

$$\cos t = -C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t$$

Collecting like terms

$$(-C + 5D + 6C)\cos t + (-D - 5C + 6D)\sin t = \cos t$$

$$(5C + 5D)\cos t + (5D - 5C)\sin t = \cos t$$

equating like terms

$$5C + 5D = 1 \quad \text{..... Eqn 1}$$

$$5D - 5C = 0 \quad \text{..... Eqn 2}$$

solving simultaneously

adding equ 1 to 2

$10D = 1$

$D = \frac{1}{10}$

∴ putting D in equ 2

$5\left(\frac{1}{10}\right) - 5C = 0$

$\frac{5}{10} = 5C \quad \therefore \frac{1}{2} = 5C$

$C = \frac{1}{10}$

P.I ∴ $x = \frac{1}{10}\cos t + \frac{1}{10}\sin t$

General Solution $Cf + P.I$

$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10}(\cos t + \frac{1}{10}\sin t)$

Given $t = 0 \quad x = 0.1 \quad dx/dt = 0$

$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - \frac{1}{10}\sin t + \frac{1}{10}\cos t$

using the first Condition when $t = 0 \; dx/dt = 0$

$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} - \frac{1}{10}\sin(0) + \frac{1}{10}(\cos(0))$

$0 = -2A - 3B - 0 + \frac{1}{10}$

$0 = -2A - 3B + 1/10 \quad \therefore -2A - 3B = -1/10$ equ 3

using the Second Condition when $t = 0 \; x = 0.1$

$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10}\cos(0) + \frac{1}{10}\sin(0)$

$0.1 = A + B + \frac{1}{10}$

$0.1 - \frac{1}{10} = A + B$

$A = -B$ equ 4

$0.1 - 0.1 = A + B$

$-2A - 3B = \frac{-1}{10}$ ... equ 3

$A = -B$ ... equ 4

Putting equ 4 into 3

$-2(-B) - 3B = \frac{-1}{10}$

$2B - 3B = -1/10$

$-B = -1/10 \therefore B = 1/10$

putting $B = 1/10$ into equ 4

$A = -1/10$

General Solution:

$x = \frac{-1}{10}e^{-2t} + \frac{1}{10}e^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$x = \frac{1}{10}\left(-e^{-2t} + e^{-3t} + \cos t + \sin t\right)$

iii Amplitude, $k = 0.1\sqrt{}$

$A \sin(\omega t + \phi)$

Period, $T = 8.55 - 2.40 = 6.15$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{6.15}$

$= 1$ rad/s or $\pi/3$ rad

a = the value of t for when $\sin t = \cos t$

$a = 45°$ or $\pi/4$ rad

Comparing with $k \sin(t + a)$

The steady state solution is given as

$0.4 \sin(t + 45°)$

$\frac{\sqrt{2}}{10} \sin\left(t + \pi/4\right)$.