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161ENG04050
Elect/Elect
ENG 381 Assignment

i $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$
 $t=0, x=0.1$ and $\frac{dx}{dt} = 0$

Taking $\cos t = 0$

~~$\frac{d^2x}{dt^2}$~~ $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$

$x = Ae^{kt}$

$\frac{dx}{dt} = kAe^{kt} = kx$

$\frac{d^2x}{dt^2} = k^2Ae^{kt} = k^2x$

Substitute for $\frac{dx}{dt}$, $\frac{d^2x}{dt^2}$ and x
 ~~k^2x~~

$k^2x + 5kx + 6x = 0$

Divide through by x

$k^2 + 5k + 6 = 0 \rightarrow$ Auxillary, Solution

$k^2 + 3k + 2k + 6 = 0$

$k(k+3) + 2(k+3) = 0$

$(k+3)(k+2) = 0$

$k = -3$ or -2

$x = Ae^{-3t} + Be^{-2t} \rightarrow$ General solution

Taking the standard form of the RHS

$x = A \cos t + B \sin t$

$\frac{dx}{dt} = -A \sin t + B \cos t$

$\frac{d^2x}{dt^2} = -A \cos t - B \sin t$

Equate the
Substitute for $\frac{dx}{dt}$, $\frac{d^2x}{dt^2}$ and x

$$[-A \cos t - B \sin t] + 5[-A \sin t + B \cos t] + 6[A \cos t + B \sin t] = \cos t$$

$$-A \cos t - B \sin t - 5A \sin t + 5B \cos t + 6A \cos t + 6B \sin t = \cos t$$

Equate the coefficients

$$-5A + 5B = 0 \quad \text{--- (i)}$$

$$5A + 5B = 1 \quad \text{--- (ii)}$$

$$-10A = -1$$

$$A = \frac{-1}{-10}$$

$$A = \frac{1}{10}$$

$$-5A + 5B = 0$$

$$5B = 5A$$

$$B = \frac{5A}{5}$$

$$B = A = \frac{1}{10}$$

Particular Integral $\rightarrow x = \frac{1}{10} \cos t + \frac{1}{10} \sin t$

$$x = \frac{1}{10} [\cos t + \sin t]$$

Recall

General solution $\rightarrow x = Ae^{-3t} + Be^{-2t}$

When $t = 0$, $x = 0.1$ and $\frac{dx}{dt} = 0$

Substituting x and t

$$0.1 = Ae^{-3(0)} + Be^{-2(0)}$$

$$0.1 = A + B \quad \text{--- (i)}$$

Substituting $\frac{dx}{dt}$ and $t = 0$

$$0 = -3Ae^{-3t} - 2Be^{-2t}$$

$$0 = -3A - 2B \quad \text{--- (ii)}$$

$$A = 0.1 - B \quad \text{--- (iii)}$$

$$0 = -3(0.1 - B) - 2B$$

$$0 = -0.3 + 3B - 2B$$

$$0.3 = 3B - 2B$$

$$B = 0.3$$

From eqn (iii) $A = 0.1 - B$

$$A = 0.1 - 0.3$$

$$A = -0.2$$

General solution $\rightarrow x = -0.2e^{-3t} + 0.3e^{-2t}$

Complete general solution

$$x = -0.2e^{-3t} + 0.3e^{-2t} + \frac{1}{10} [\cos t + \sin t]$$

ii Command window

clear

clc

syms t

$$x = -0.2 * \exp(-3 * t) + 0.3 * \exp(-2 * t) + 0.1 * \cos(t) + 0.1 * \sin(t)$$

$$t_n = [0:0.01:15]$$

$$x_n = \text{subs}(x, t_n)$$

figure(1) figure(1)

plot(tn, xn)

xlabel('distance')

ylabel('time')

axis tight

grid on

grid minor

iii Steady state equation

$$x = \frac{\sqrt{2}}{10} \sin \left(t + \frac{\pi}{a} \right)$$