Assignment

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MATRIC 16/ENG06/013

Mechanical Engineering / 300L.

1. $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

$t = 0$, $x = 0.1$ and $\frac{dx}{dt} = 0$

taking $\cos t = 0$.

$\rightarrow \frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$

$x = Ae^{kt}$

$\frac{dx}{dt} = kAe^{kt} = kx$.

$\frac{d^2x}{dt^2} = k^2Ae^{kt} = k^2x$

substitute for $\frac{dx}{dt}$, $\frac{d^2x}{dt^2}$ and x.

$k^2x + 5kx + 6x = 0$

divide through by x.

$k^2 + 5k + 6 = 0 \rightarrow$ auxiliary solution.

$(k^2 + 3k) + (2k + 6) = 0$

$k(k+3) + 2(k+3) = 0$

$(k+3)(k+2) = 0$.

$k = -3$ or $k = -2$.

$x = Ae^{-3t} + Be^{-2t}$ — is the general solution.

taking the standard form of th RHS.

$x = A\cos t + B\sin t$

$\frac{dx}{dt} = -A\sin t + B\cos t$.

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$$\frac{d^2x}{dt^2} = -A\cos t - B\sin t.$$

subst for $\frac{dx}{dt}$, $\frac{d^2x}{dt}$ and x.

$$[-A\cos t - B\sin t] + 5[-A\sin t + B\cos t] + 6[A\cos t + B\sin t] = \cos t.$$

$$= A\cos t - B\sin t - 5A\sin t + 5B\cos t + 6A\cos t + 6B\sin t = \cos t.$$

$$\sin t[-B - 5A + 6B] + \cos t[-A + 5B + 6A] = \cos t.$$

$$\sin t[-5A + 5B] + \cos t[5B + 5A] = \cos t.$$

Equate the coefficients.

$$-5A + 5B = 0 \quad \text{—} \quad (I)$$

$$-5A + 5B = 1 \quad (II).$$

$$-10A = -1$$

$$A = \frac{-1}{-10}$$

$$A = \frac{1}{10}$$

$$-5A + 5B = 0$$

$$5B = 5A$$

$$B = \frac{5A}{5}$$

$$B = A = \frac{1}{10}.$$

Particular integral $\longrightarrow$ $x = \frac{1}{10}\cos t + \frac{1}{10}\sin t.$

$$x = \frac{1}{10}[\cos t + \sin t].$$

Recall : General solution $\rightarrow x = Ae^{-3t} + Be^{-2t}.$

When $t = 0$; $x = 0.1$ and $\frac{dx}{dt} = 0$ $\quad \frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t}.$

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Substituting x and t:

$$0.1 = Ae^{-3(0)} + Be^{-2(0)}$$

$$0.1 = A + B \quad \text{---} (i)$$

substituting $\frac{dx}{dt}$ and $t = 0$:

$$0 = -3Ae^{-3(0)} - 2Be^{-2(0)}$$

$$0 = -3A - 2B \quad \text{---} (ii)$$

$$A = 0.1 - B \quad \text{---} \; iii$$

$$0 = -3(0.1 - B) - 2B$$

$$0 = -0.3 + 3B - 2B$$

$$0.3 = 3B - 2B$$

$$B = 0.3$$

from eqn (iii) $A = 0.1 - B$

$$A = 0.1 - 0.3$$

$$A = -0.2$$

General solution $\rightarrow$ $x = 0.2e^{-3t} + 0.3e^{-2t}$

Compute general solution $\rightarrow$ $\cancel{x = 0.2e^{-3t} + 0.3e^{-2t} +}$

$$x = 0.2e^{-3t} + 0.3e^{-2t} + \frac{1}{10}(\cos t + \sin t).$$

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ii

Command Window

```
Clear
clc
syms t
x = -0.2 * exp(-3*t) + 0.3* exp(-2 t) + 0.1*cos(t) + 0.1*sin(t)
tn = [0:0.01:15]
xn = subs (x, tn)
figure (1)
Plot (tn, xn)
x label ('distance')
y label ('time')
axis tight
grid on
grid minor
```

Steady State equation

$$x = \frac{\sqrt{2}}{10} \sin\left(t + \frac{\pi}{4}\right)$$

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