

Cedric Egwoba

16/SCIO1/010

ENG 381

ELECT - ELECT

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t \quad \text{--- (i)}$$

Assume $f(x) = \cos t = 0$

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$$

Auxiliary equation; $m^2 + 5m + 6 = 0$

$$m^2 + 3m + 2m + 6 = 0$$

$$m(m+3) + 2(m+3) = 0$$

$$(m+2)(m+3) = 0$$

$$m+2 = 0 \quad \text{or} \quad m+3 = 0$$

$$m_1 = -2, \quad m_2 = -3$$

Complementary Function (CF) = $y = Ae^{-2t} + Be^{-3t}$

Particular integral (PI) = $f(x) = \cos t$

$$\therefore x = C \cos t + D \sin t \quad \text{--- (i)}$$

$$\frac{dx}{dt} = -C \sin t + D \cos t \quad \text{--- (ii)}$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t \quad \text{--- (iii)}$$

input (ii), (iii), and (iv) into equation (i)

$$\therefore -C \cos t - D \sin t + 5(-C \sin t + D \cos t)$$

$$+ 6(C \cos t + D \sin t) = \cos t$$

$$-C \cos t - D \sin t - 5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t = \cos t$$

$$-C \cos t + 5D \cos t + 6C \cos t - D \sin t - 5C \sin t + 6D \sin t = \cos t$$

So

$$\cos t (-C + 5D + 6C) + \sin t (-D - 5C + 6D) = \cos t$$

$$10C = 1$$

$$C = 1/10 \quad \text{--- (vii)}$$

sub eqn (vii) into eqn (v)

$$5\left(\frac{1}{10}\right) + 5D = 1$$

$$\frac{1}{2} + 5D = 1$$

$$5D = 1 - \frac{1}{2}$$

$$5D = \frac{1}{2}$$

$$D = \frac{1}{2} \times \frac{1}{5}$$

$$D = \frac{1}{10}$$

$$P.I = x = \frac{\cos t}{10} + \frac{\sin t}{10}$$

$$\begin{aligned} \text{General solution} &= C.F + P.I \\ &= x = A e^{-2t} + B e^{-3t} + \frac{\cos t}{10} + \frac{\sin t}{10} \end{aligned}$$

$$\Rightarrow \text{at } t=0, \quad x=0.1 \quad \text{and} \quad \frac{dx}{dt} = 0$$

$$0.1 = A e^{-2(0)} + B e^{-3(0)} + \frac{\cos(0)}{10} + \frac{\sin(0)}{10}$$

$$0.1 = A + B + \frac{1}{10} + 0$$

$$A + B = 0.1 - \frac{1}{10}$$

$$\frac{dx}{dt} = -2A e^{-2t} - 3B e^{-3t} - \frac{\sin t}{\omega} + \frac{\omega t}{\omega}$$

input $\frac{dx}{dt} = 0, t = 0$

$$0 = -2A e^{-2t} - 3B e^{-3t} - \frac{\sin t}{\omega} + \frac{\omega t}{\omega}$$

$$0 = -2A e^{-2(0)} - 3B e^{-3(0)} - \frac{\sin(0)}{\omega} + \frac{\omega(0)}{\omega}$$

$$0 = -2A(1) - 3B(1) - 0 + 1/\omega$$

$$2A + 3B = \frac{1}{\omega} \quad \text{--- (ix)}$$

$$A + B = 0 \quad \text{--- (viii)}$$

$$2A + 3B = \frac{1}{\omega} \quad \text{--- (ix)}$$

from eqn (viii) $A + B = 0$,
 $A = -B$ --- (x)

Sub eqn (x) into (ix)

$$2(-B) + 3B = \frac{1}{\omega}$$

$$-2B + 3B = \frac{1}{\omega}$$

$$B = \frac{1}{\omega}$$

$$x = \frac{1}{10} (e^{-2t} + e^{-3t}) (\cos t + \sin t)$$