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16/ENG05/107
MECHATRONICS ENGINEERING
300Level

Assignment

1a $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

Solution

$m^2 + 5m + 6m = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-5 \pm \sqrt{5^2 - 4(1)(6)}}{2(1)}$

$= \frac{-5 \pm \sqrt{25 - 24}}{2}$

$= \frac{-5+1}{2}$ or $\frac{-5-1}{2}$

$= -4/2, -6/2$

$= -2, -3$

C.F $\Rightarrow x = Ae^{-2t} + Be^{-3t}$

P.I

$x = C\cos t + D\sin t$

$\frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

$= -C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$= -C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

$= C - (+5D + 6C)\cos t + (-D - 5C + 6D)\sin t = \cos t$

Equating Coefficient

$-C + 5D + 6C = 1$

$5C + 5D = 1 \quad -(1)$

$-D - 5C + 6D = 0$

$-5C + 5D = 0 \quad -(2)$

$-5C + 5D = 1$

$-5C + 5D = 0$

$10C = 1$

$C = 1/10$

$5(1/10) + 5D = 1$

$1/2 + 5D = 1$

$5D = 1 - 1/2$

$D = 1/10$

$x = 1/10 \cos t + 1/10 \sin t$

$= C.F + P.I$

$= 4e^{-2t} + Be^{-3t} + 1/10 \cos t + 1/10 \sin t$

(3) $x = \frac{1}{10}(-e^{-2t} + e^{3t}) + (\cos t + \sin t)$

The transient part of the equation will not be considered since we are dealing with the steady state

$x = \frac{1}{10}(\cos t + \sin t)$

we know $\frac{dx}{dt} = 0$ for steady state

$$\frac{dx}{dt} = \frac{1}{10}(-\sin t + \cos t) = 0$$

$$\therefore -\sin t + \cos t = 0$$

$$-\cos t = \sin t$$

$$t = 45^0$$

$$\therefore x = \frac{1}{10}(\cos 45 + \sin 45)$$

$$x = \sqrt{2}/10$$

from sinosoidal expression

$$A\cos\omega + P\sin\omega = A\cos\omega t - 0$$

But; $\cos(\omega t - \theta) = \sin(\omega t - \theta + 90^\circ)$

where $k = \sqrt{A^2+B^2} = \sqrt{(1/10)^2+(1/10)^2}$

$$\frac{\sqrt{1+1}}{10}$$

$$k = \frac{\sqrt{2}}{10}$$

$\theta = 0^\circ$ since its in the same phase

Recall $x = k\sin(t+a)$

$$\frac{\sqrt{2}}{10} = \frac{\sqrt{2}}{10}\sin(45^\circ + a)$$

$$a = 90^\circ - 45^\circ$$

$$= 45^0$$

$$= \pi/4$$

Steady State Solution

$$x = \frac{\sqrt{2}}{10}(\sin t + \pi/4)$$