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16/ENG06/059

Mechanical Engineering

8001

ENG 381

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

$$m^2 + 5m + 6 = 0$$

$$m^2 + 2m + 3m + 6 = 0$$

$$m(m+2) + 3(m+2) = 0$$

$$\therefore m = -2, -3$$

$$\text{from } x_c = Ae^{m_1 t} + Be^{m_2 t}$$

$$x_c = Ae^{2t} + Be^{-3t}$$

$$\text{for P.I, } x = \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$(-C \cos t - D \sin t) + 5(-C \sin t + D \cos t)$$

$$+ 6(C \cos t + D \sin t) = \cos t$$

$$-C \cos t - D \sin t - 5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t = \cos t$$

$$6C \cos t + 6D \sin t = \cos t$$

$$5C + 5D = 1; 5D - 5C = 0$$

$$5C + 5D = 1$$

$$-5C + 5D = 0$$

$$10C = 1; C = \frac{1}{10}$$

$$5\left(\frac{1}{10}\right) + 5D = 1$$

$$\frac{5}{10} + 5D = 1$$

$$; D = \frac{1}{10}; x_c = \frac{1}{10}e^{2t} + \frac{1}{10}e^{-3t}$$

$$x = \frac{1}{10}(\cos t + \sin t)$$

$$\therefore x = Ae^{-2t} + Be^{3t} + \frac{1}{10}(\cos t + \sin t)$$

$$\therefore dx = -2Ae^{-2t} - 3Be^{3t} + \frac{1}{10}$$

$$(-5 \sin t + \cos t) \text{ eqn(2)}$$

from eqn(1)

$$\text{given } t=0, x=0, \frac{dx}{dt} = 0$$

$$0 = A + B + 0.1 \text{ (1)}$$

$$\therefore A + B = 0 \dots \text{eqn(2)}$$

from eqn(2)

$$-2A - 3B + 0.1 = 0$$

$$-2A - 3B = 0.1 \dots \text{eqn(4)}$$

$$A = -B$$

$$-2(-B) - 3B = 0.1$$

$$+ 2B - 3B = 0.1$$

$$-B = 0.1 \therefore B = 0.1$$

$$A = 0.1$$

$$\therefore x = 0.1x e^{-2t} - 0.1x e^{-3t} +$$

$$0.1(\cos t + \sin t)$$

$$x = 0.1(Ce^{2t}e^{-2t} + \cos t + \sin t)$$

$$x = \frac{1}{10}(e^{2t}e^{-2t} + \cos t + \sin t)$$

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ii) MATLAB program to plot the relationship between x and t for $0 \leq t \leq 15$ Unit a Step Size of 0.1 Unit

Soln

1. Command window

- Clear

- clc

- Close all

- Syms

$x = 0.1 * (\exp(-3*t) - \exp(-2*t)) + \cos(t) + \sin(t)$

- $tn = [0; 0.1; 0.1; 15]$

- $xn = \text{subs}(x, tn)$

- figure(1)

- plot(tn, xn)

- grid on

- grid minor

- axis tight

- xlabel('t')

- ylabel('x')

Figure 1

