





Documents > MATLAB

Editor - C:\Users\OCHENI VICTOR\Documents\MATLAB\mid.m

```
mid.m x +
1 -  commandwindow
2 -  clear
3 -  clc
4 -  close all
5 -  syms t
6 -  x = (0.1) * (-exp(-2*t) + exp(-3*t) + cos(1*t) + sin(1*t))
7 -  ts = [0:0.01:15]
8 -  xs = subs(x, ts)
9 -  figure(1)
10 - plot(ts, xs)
11 - xlabel('Time (seconds)')
12 - ylabel('Vibrations')
13 - grid on
14 - grid minor
15 - axis tight
```

Command Window

```
14.9400 14.9500 14.9600 14.9700 14.9800 14.9900 15.0000

xs =
[ 1/10, cos(1/100)/10 - exp(-1/50)/10 + exp(-3/100)/10 + sin(1/100)/10, cos(1/50)/10 - exp(-1/25)/10 + exp(-3/50)/10 + sin(1/50)/10, ...
ft >>
<
```

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

$$\text{let } x = e^{kt}$$

$$x' = ke^{kt} = kx$$

$$x'' = k^2e^{kt} = k^2x$$

Complementary function

$$k^2x + 5kx + 6x = 0$$

$$x(k^2 + 5k + 6) = 0$$

$$k^2 + 5k + 6 = 0$$

$$(k+2)(k+3) = 0$$

$$k_1 = -2, k_2 = -3$$

$$\therefore x = Ae^{-2t} + Be^{-3t}$$

particular integral

$\cos t$

general form $x = A\cos t + B\sin t$

$$x' = -A\sin t + B\cos t$$

$$x'' = -A\cos t - B\sin t$$

$$-(A\cos t + B\sin t) + 5(-A\sin t + B\cos t) + 6(A\cos t + B\sin t) = \cos t$$

$$-A\cos t - B\sin t - 5A\sin t + 5B\cos t + 6A\cos t + 6B\sin t = \cos t$$

$$5A\cos t + 5B\sin t - 5A\sin t + 5B\cos t = \cos t$$

$$\cos t(5A + 5B) + \sin t(5B - 5A) = \cos t + 0\sin t$$

$$5A + 5B = 1$$

$$-5A + 5B = 0$$

$$10B = 1$$

$$B = \frac{1}{10}$$

$$\text{or } 5A + 5B = 1$$

$$5A + 5\left(\frac{1}{10}\right) = 1$$

$$5A + \frac{1}{2} = 1$$

$$5A = \frac{1}{2}$$

$$A = \frac{1}{10}$$

$$\therefore a = -\frac{1}{10}, b = \frac{1}{10}$$

$$x = Ae^{-2t} + Be^{-3t} - \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

When $t=0$, $x=0$, $\frac{dx}{dt}=0$

$$x = Ae^{-2t} + Be^{-3t} - \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$\frac{1}{10} = Ae^{-2(0)} + Be^{-3(0)} - \frac{1}{10} \cos 0 + \frac{1}{10} \sin 0$$

$$\frac{1}{10} = A + B - \frac{1}{10}$$

$$A + B = \frac{2}{10} \quad \dots \text{ (i)}$$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} + \frac{1}{10} \sin t - \frac{1}{10} \cos t$$

$$0 = -2Ae^0 - 3Be^0 + \frac{1}{10} \sin 0 - \frac{1}{10} \cos 0$$

$$-2A - 3B = -\frac{1}{10}$$

$$2A + 3B = \frac{1}{10} \quad \dots \text{ (ii)}$$

$$(A + B = \frac{2}{10}) \times 2$$

$$2A + 2B = \frac{4}{10}$$

$$2A + 2B = \frac{4}{10}$$

$$-2A + 3B = \frac{1}{10}$$

$$-B = \frac{3}{10}$$

$$B = -\frac{3}{10}$$

$$A + B = \frac{2}{10}$$

$$A - \frac{3}{10} = \frac{2}{10}$$

$$A = \frac{5}{10} = \frac{1}{2}$$

\therefore at $t=0$, $x=0$ and $\frac{dx}{dt}=0$

$$x = \frac{1}{2}e^{-2t} - \frac{3}{10}e^{-3t} - \frac{1}{10} \cos t + \frac{1}{10} \sin t.$$

$$(3) x = \frac{1}{10} (-e^{-2t} + e^{-3t}) + (\cos t + \sin t)$$

since $\frac{dx}{dt} = 0$ for a steady state

$$\frac{dx}{dt} = \frac{1}{10} (-\sin t + \cos t) = 0$$

$$-\sin t + \cos t = 0$$

$$\cos t = \sin t$$

$$\therefore t = 45^\circ$$

$$x = \frac{1}{10} (\cos 45 + \sin 45) = \frac{\sqrt{2}}{10}$$

From the sinusoidal expression

$$A \cos \omega t + B \sin \omega t + K \cos(\omega t - \theta)$$

$$\text{but } \cos(\omega t - \theta) = \sin(\omega t - \theta + 90^\circ)$$

where

$$K = \sqrt{A^2 + B^2} = \sqrt{(\frac{1}{10})^2 + (\frac{1}{10})^2}$$

$$= \sqrt{\frac{1}{10^2}}$$

$$K = \frac{\sqrt{2}}{10}$$

$$\theta = 0^\circ$$

$$\text{Recall } x = \frac{K \sin(1 + a)}{\sqrt{2}}$$

$$K = \frac{\sqrt{2}}{10} \sin 45 + a$$

$$1 = \sin(45 + a)$$

$$45 + a = \sin^{-1}(1)$$

$$a = 90 - 45 = 45^\circ = \frac{\pi}{4}$$

The steady state solution

$$x = \frac{\sqrt{2}}{10} \sin(t + \frac{\pi}{4})$$