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16/ENG04
Electrical Engineering
ENG 381

SOLUTION:

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Auxiliary equation $= m + 5m + 6$

$m + 5m + 6$

$(m+2)(m+3)$

$m_1 = -2, \quad m_2 = -3$

$C.F: \; y = Ae^{-2x} + Be^{-3x}$

$P.I \Rightarrow y = (C\cos t + D\sin t)$

$$\frac{dy}{dt} = -(\sin t + D\cos t)$$

$$\frac{d^2y}{dt^2} = -(C\cos t - D\sin t)$$

Substitute $\frac{dy}{dt}$ and $\frac{d^2y}{dt^2}$ into original equation

$[-(C\cos t - D\sin t)] + 5[-(\sin t + D\cos t)] + 6[(C\cos t + D\sin t)] = \cos t$

$-(C\cos t - D\sin t) - 5(\sin t + D\cos t) + 6C\cos t + 6D\sin t$
$\cos t$

$\cos t[-C + 5D + 6C] + \sin t[-D - 5C + 6D] = \cos t$

$\cos t[5D + 5C] - \sin t[-5C + 5D] = \cos t$

Solution

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$$\frac{d^2y}{dt^2} = -C\cos t - D\sin t$$

Sub $\frac{dy}{dt}$ and $\frac{d^2y}{dt^2}$ into original eqn

$[-C\cos t - D\sin t] + 5[-C\sin t + D\cos t] + C[C\cos t + D\sin t] = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t - \cos t$

$\cos t[-C + 5D + 6C] + \sin t[-D - 5C + 6D] = \cos t$

$\cos t[5D + 5C] - \sin t[-5C + 5D] = \cos t$

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Comparing coefficient.

$\cos t$: $5D + 5C = 1$

$\sin t$: $-5C + 6D = 0$

$\cos t$: $5D + 5C = 1$ - - - (1)

$\sin t$ $5D - 5C = 0$ - - - (2)

Solving simultaneously.

$$\begin{array}{r} 5D + 5C = 1 \\ 5D - 5C = 0 \\ \hline 10D - 0 = 1 \end{array}$$

$10D = 1$

$D = \frac{1}{10}$

input $D = \frac{1}{10}$ in eqn (2)

$5D - 5C = 0$

$5\left(\frac{1}{10}\right) - 5C = 0$

$\frac{1}{2} - 5C = 0$

$5C = \frac{1}{2}$

$C = \frac{1}{2} \times \frac{1}{5}$

$C = \frac{1}{10}$

$C = \frac{1}{10}$, $D = \frac{1}{10}$ $\therefore$ $P.I = \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$G.S = C.F + P.I$

$y = Ae^{-2t} + Be^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

using the values of $t = 0$, $x = 0.1$, $\frac{dx}{dt} = 0$

$y = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10}\cos(0) + \frac{1}{10}\sin(0)$

$0.1 = A + B + \frac{1}{10} + 0$

$0.1 = A + B + \frac{1}{10}$

$0.1 - \frac{1}{10} = A + B$

$A + B = 0$

$A = -B$

$\frac{dx}{dt} = 2Ae^{-2t} + 3Be^{-3t} - \frac{1}{10}\sin t + \frac{1}{10}\cos t$

$\therefore \frac{dx}{dt} = -2Be^{-2t} + 3Be^{-3t} - \frac{1}{10}\sin t + \frac{1}{10}\cos t$

$0 = -2B + 3B - 0 + \frac{1}{10}$

$0 = B + \frac{1}{10}$

$B = -0.1$

$A = 0.1$

$y = 0.1e^{-2t} - 0.1e^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$y = 0.1e^{-2t} - 0.1e^{-3t} + \frac{1}{10}(\cos t + \sin t).$

iii) $x = K\sin(t + a)$

$\therefore x = \frac{1}{10}\left[(6 - e^{-2t} + e^{-3t}) + (\cos t + \sin t)\right]$

using steady part of the equation.

$$x = \frac{1}{10}(\cos t + \sin t)$$

Note: $\frac{dx}{dt} = 0$ for steady state

$$\frac{dx}{dt} = \frac{1}{10}(-\sin t + \cos t) = 0$$

$$-\sin t + \cos t = 0$$

$$\cos t = \sin t$$

Hence: $t = 45^\circ$ or $\pi/4$

$$x = \frac{1}{10}(\cos 45 + \sin 45)$$

Sinusoidal equation: $A\cos\omega t + B\sin\omega t = K\cos(\omega t - \theta)$

but $\cos(\omega t - \theta) = \sin(\omega t - \theta + 90^\circ)$

where: $K = \sqrt{A^2 + B^2} = \sqrt{(1/10)^2 + (1/10)^2} = 0.14$

Since it is on the same phase, hence: $\theta = 0^\circ$

$$x = \frac{1}{10}(\cos 45 + \sin 45)$$

$$= 0.14\sin(45 + 90^\circ)$$

$$x = 0.14\sin(45 + 90^\circ)$$

$$x = 0.14\sin(90 + 45)_{\shortparallel}$$

in terms of the above equation, Hence

$$K = 0.14 \text{ or } \frac{\sqrt{2}}{10}$$

$$a = 45 \text{ or } \frac{\pi}{2}$$

$$x = 0.14\sin(t + 45)_{\shortparallel}$$