Malik Abdulrasheed

16/ENG05/022 300L

Mechatronics

ENG 381

(1) $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

let $x = e^{kt}$

$x' = ke^{kt} = kx$

$x'' = k^2e^{kt} = kx$

complementary function

$k^2x + 5kx + 6x = 0$

$x(k^2+5k+6) = 0$

$k^2+5k+6 = 0$

$(k+2)(k+3) = 0$

$k_1 = -2$, $k_2 = -3$

$\therefore x = Ae^{-2t} + Be^{-3t}$

particular integral

cost

general form $x = A\cos t + B\sin t$

$x' = -a\sin t + b\cos t$

$x'' = -a\cos t - b\sin t$

$-(a\cos t + b\sin t) + 5(-a\sin t + b\cos t) + 6(a\cos t + b\sin t) = \cos t$

$-a\cos t - b\sin t - 5a\sin t + 5b\cos t + 6a\cos t + 6b\sin t = \cos t$

$5a\cos t + 5b\sin t - 5a\sin t + 5b\cos t = \cos t$

$\cos t(5a+5b) + \sin t(5b-5a) = \cos t + 0\sin t$

$5a + 5b = 1$

$+ \quad -5a + 5b = 0$

$10b = 1$

$b = 1/10$

$5a+5b = 1$

$5a + 5(1/10) = 1$

$5a + \frac{1}{2} = 1$

$5a = -\frac{1}{2}$

$a = -\frac{1}{10}$

$\therefore a = -\frac{1}{10}, \quad b = \frac{1}{10}$

$x = Ae^{-2t} + Be^{-3t} - \frac{1}{10}\cos t + \frac{1}{10}\sin t$

when $t = 0, \; x = 0, \; \frac{dx}{dt} = 0$

$x = Ae^{-2t} + Be^{-3t} - \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$\frac{1}{10} = Ae^{-2(0)} + Be^{-3(0)} - \frac{1}{10}\cos 0 + \frac{1}{10}\sin 0$

$\frac{1}{10} = A + B - \frac{1}{10}$

$A + B = \frac{2}{10} \quad — (1)$

$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} + \frac{1}{10}\sin t - \frac{1}{10}\cos t$

$0 = -2Ae^{0} - 3Be^{0} + \frac{1}{10}\sin 0 + \frac{1}{10}\cos 0$

$-2A - 3B = -\frac{1}{10}$

$2A + 3B = \frac{1}{10} \quad — (11)$

$(A + B = \frac{2}{10}) \times 2$

$2A + 3B = \frac{1}{10}$

$-\;2A + 2B = \frac{4}{10}$

$-B = \frac{3}{10}$

$B = -\frac{3}{10}$

$A + B = \frac{2}{10}$

$A - \frac{3}{10} = \frac{2}{10}$

$A = \frac{5}{10} = \frac{1}{2}$

$\therefore$ at ~~to~~ $t = 0, \; x = 0.1$ and $\frac{dx}{dt} = 0$

$x = \frac{1}{2}e^{-2t} - \frac{3}{10}e^{-3t} - \frac{1}{10}\cos t + \frac{1}{10}\sin t$

Scanned by CamScanner

(3) $x = \frac{r}{10}\left(-e^{-2t} + e^{-3t}\right) + (\cos t + \sin t)$

since $\frac{dx}{dt} = 0$ for a steady state

$\frac{dx}{dt} = \frac{1}{10}(-\sin t + \cos t) = 0$

$-\sin t + \cos t = 0$

$\cos t = \sin t$

$\therefore t = 45^0$

$x = \frac{1}{10}(\cos 45 + \sin 45) = \frac{\sqrt{2}}{10}$

from the sinosoidal expression

$A\cos\omega t + B\sin\omega t + K\cos(\omega t - \theta)$

but $\cos(\omega t - \theta) = \sin(\omega t - \theta + 90)$

where

$k = \sqrt{A^2 + B^2} = \sqrt{(1/10)^2 + (1/10)^2}$

$= \sqrt{{}^{111}/10^2}$

$k = \sqrt{2}/10$

$\theta = 0^0$

Recall $\frac{x}{\sqrt{2}} = k\sin(1 + a)$

$k = \frac{\sqrt{2}}{10}\sin 45 + a$

$1 = \sin(45 + a)$

$45 + a = \sin^{-1}(1)$

$a = 90 - 45 = 45^0 = \pi/4$

The steady state solution

$x = \frac{\sqrt{2}}{10}\sin(t + \pi/4)$

Scanned by CamScanner