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ENG 381

GABRIEL ABDULMALIK KASIM

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CIVIL ENG

ASSIGNMENT 1

$$1 \quad \frac{d^2x}{dt^2} + 5 \frac{dx}{dt} + 6x = \cos t \quad \text{--- (1)}$$

$$x = C.F + P.I$$

General solution $x = C.F + P.I$ Assume that $C.F = 0$

$$\therefore \frac{d^2x}{dt^2} + 5 \frac{dx}{dt} + 6 = 0$$

$$m^2 + 5m + 6 = 0$$

$$m = \frac{-5 \pm \sqrt{5^2 - 4(1)(6)}}{2(1)}$$

$$= \frac{-5 \pm \sqrt{25 - 24}}{2}$$

$$= \frac{-5 \pm 1}{2}$$

$$= \frac{-5 \pm 1}{2}$$

$$m = -2 \text{ or } -3$$

$$x_c = Ae^{-2t} + Be^{-3t} \quad (C.F)$$

$$P.I: x_p = C \cos t + D \sin t$$

$$\frac{dx_p}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x_p}{dt^2} = -C \cos t - D \sin t$$

Sub x_p , $\frac{dx_p}{dt}$ and $\frac{d^2x_p}{dt^2}$ in eqn (1)

$$(-C \cos t - D \sin t) + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t$$

$$-C \cos t - D \sin t - 5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t = \cos t$$

$$(-C + 5D + 6C) \cos t + (-D - 5C + 6D) \sin t = \cos t$$

$$(5C + 5D) \cos t + (-5C + 5D) \sin t = \cos t$$

$$5C + 5\Delta = 1 \quad \text{--- (2)}$$

$$+ \quad -5C + 5\Delta = 0 \quad \text{--- (3)}$$

$$10\Delta = 1$$

$$\Delta = \frac{1}{10}$$

$$\text{Sub } \Delta = \frac{1}{10} \text{ in (2)}$$

$$5C + 5\left(\frac{1}{10}\right) = 1$$

$$5C + \frac{1}{2} = 1$$

$$5C = \frac{1}{2}$$

$$C = \frac{1}{10}$$

$$\text{Sub } C \text{ and } \Delta \text{ in } x_p$$

$$x_p = \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$\text{General Solution: } x = \text{C.F.} + \text{P.I.}$$

$$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$\text{When } t=0, x=0.1$$

$$\therefore 0.1 = Ae^{-2 \times 0} + Be^{-3 \times 0} + \frac{1}{10} \cos 0 + \frac{1}{10} \sin 0$$

$$0.1 = A + B + \frac{1}{10} \quad \text{--- (4)}; \quad A + B = 0 \quad \text{--- (5)}$$

$$\text{When } t=0, \frac{dx}{dt} = 0$$

$$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - \frac{1}{10} \sin t + \frac{1}{10} \cos t$$

$$\text{When } t=0, \frac{dx}{dt} = 0$$

$$\therefore 0 = -2Ae^{-2 \times 0} - 3Be^{-3 \times 0} - \frac{1}{10} \sin 0 + \frac{1}{10} \cos 0$$

$$0 = -2A - 3B + \frac{1}{10} \quad \text{--- (6)}$$

$$A + B = 0 \quad - \textcircled{4} \times 2$$

$$2A + 3B = \frac{1}{10} \quad - \textcircled{5} \times 1$$

$$2A + 2B = 0 \quad -$$

$$\underline{2A + 3B = \frac{1}{10}}$$

$$-B = -\frac{1}{10}$$

$$B = \frac{1}{10}$$

Sub B in eqn $\textcircled{4}$

$$A + \frac{1}{10} = 0$$

$$A = -\frac{1}{10}$$

Sub A and B in the general solution

$$\therefore x = \underline{-0.1 e^{-2t} + 0.1 e^{-3t} + 0.2 \cos t + 0.1 \sin t}$$