OTUKOYA ISMAIL

16/ENG05/025

Mechatronics

1) $\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t$

$f(x) = 0$

$m^2 + 5m + 6 = 0$

$m = \frac{-5 \pm \sqrt{5^2 - 4 \times 1 \times 6}}{2}$

$m = \frac{-5+1}{2}$ or $\frac{-5-1}{2}$

$m = -2$ or -3

$x = Ae^{-2t} + Be^{-3t}$

Particular Integral

$x = C\cos t + D\sin t$

$\frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

Substituting

$[-C\cos t - D\sin t] + 5[-C\sin t + D\cos t] + 6[C\cos t + D\sin t] = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6\cos t + 6D\sin t = \cos t$

$-C\cos t + 6C\cos t + 5D\cos t - D\sin t + 6D\sin t - 5C\sin t = \cos t$

$(5C + 5D)\cos t + (5D - 5C)\sin t = \cos t$

$5D + 5C = 1$

$5D - 5C = 0$

$10D = 1$

$D = 1/10$

$5D + 5C = 1$

$5[1/10] + 5C = 1$

$5C = 0.5$

$C = 0.5/5 = 1/10$

$x = \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10}C\cos t + \frac{1}{10}\sin t$

when $t = 0$, $x = 0.1$, $dx/dt = 0$

$0.1 = Ae^{-2(0)} + Be^{-3(0)} + 1/10\cos(0) + 1/10\sin(0)$

$0.1 = A + B + 1/10$

$A + B = 0$

$dx/dt = -2Ae^{-2t} - 3Be^{-3t} - 1/10\sin t + 1/10\cos t$

$0 = -2A - 3B + 1/10$

$2A + 3B = 1/10$

$A = -B$

$-2B + 3B = 1/10$

$B = 1/10$, $A = -1/10$

$x = \frac{-1}{10}e^{-2t} + \frac{1}{10}e^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

ii)

Command windows

Clear

Clc

Close all

Syms t

x(t) = 0.1 * ((exp(-3*t)) - ((exp(-2*t)) + (os(t) + sin(t)

tn = [0 : 0.01 : 15]

xd = Subs (x(t), tn)

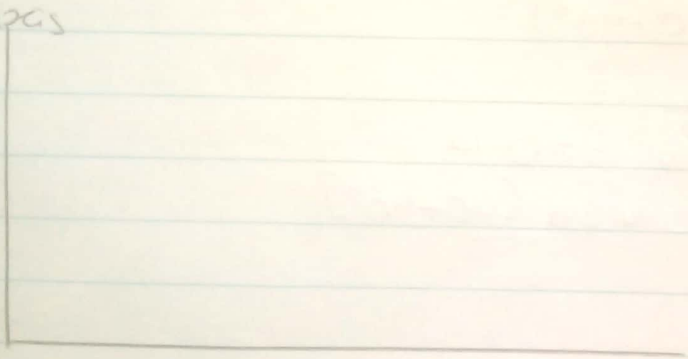
Figure (1)

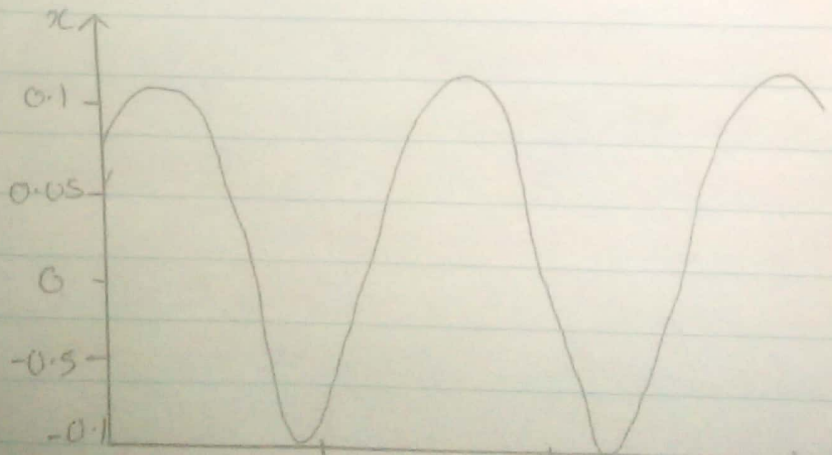
Plot (tn, xd)

grid minor

grid on

axis





iii) The steady state is $x = C\cos t + D\sin t$

$$x = \frac{1}{10}\left(-e^{-2t} + e^{-3t}\right) + \left(C\cos t + \sin t\right)$$

$$x = \frac{1}{10}\left(C\cos t + \sin t\right)$$

$$\frac{dx}{dt} = \frac{1}{10}\left(-\sin t + \cos t\right) = 0$$

$$-\sin t + \cos t = 0$$

$$\cos t = \sin t$$

$$t = 45°$$

$$x = \frac{1}{10}\left(\cos 45 + \sin 45\right)$$

Recall: $A\cos\omega t + B\sin\omega t = K\cos(\omega t - \theta)$

$\cos(\omega t - \theta) = \sin(\omega t - \theta + 90°)$

where

$$K = \sqrt{A^2 + B^2} = \sqrt{(1/10)^2 + (1/10)^2} = 0.14$$

$$x = \frac{1}{10}\left(\cos 45 + \sin 45\right) = 0.14\sin(45 + 90°)$$

$$x = 0.14\sin(45 + 90°)$$