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16/ENG02/012

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COMPUTER ENGINEERING.

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t.$$

$$m^2 + 5m + 6 = 0$$

$$(m^2 + 3m) + (2m + 6) = 0$$

$$m(m + 3) + 2(m + 3) = 0$$

$$(m + 2)(m + 3) = 0$$

$$m = -2 \text{ or } m = -3.$$

$$C.F = Ae^{-2t} + Be^{-3t}$$

$$x = C \cos t + D \sin t.$$

$$\frac{dx}{dt} = -C \sin t + D \cos t.$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t.$$

$$- (C \cos t - D \sin t) + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) =$$

$$- (C \cos t - D \sin t - 5(C \sin t - D \cos t) + 6C \cos t + 6D \sin t) = \cos t$$

$$(-C + 5D + 6C) \cos t + (-D - 5C + 6D) \sin t = \cos t.$$

$$-C + 5D + 6C = 1$$

$$5C + 5D = 0$$

$$-D - 5C + 6D = 0$$

$$10C = 1$$

$$C = \frac{1}{10} = 0.1$$

$$5C + 5D = 0$$

$$\frac{1}{2} + 5D = 0$$

$$5D = -\frac{1}{2}$$

$$5D = -\frac{1}{2}$$

$$D = -\frac{1}{10}$$

$$G.S = y = C.F + P.I$$

$$= Ae^{-2t} + Be^{-3t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t.$$

ii) Write a mat lab program to plot the relationship x if the

Solution:

- Command window
- Clear.
- clc
- Close all
- Sysns 2.

$$x = 0.1x(\exp(-3t) - \exp(-2t)) + (\cos t) + \sin$$

$$t_n = (0.001:0.15)$$

$$f_n = \text{subs}($$

Figure 1.

plot (t_n, x_n).

Axis tight

xlabel ('time')

ylabel ('ibrahim')

iii) Write the steady solution of the system in

Solution

$$x = \frac{1}{10} (e^{-3t} - e^{-2t} + \sin t + \cos t)$$

steady state $\frac{\partial x}{\partial t} = 0$ i.e.

$$\frac{\partial x}{\partial t} = \frac{1}{10} (-3e^{-3t} - e^{-2t} + \cos t - 3\sin t)$$

; the exponential result shows

$$0 = \cos t - \sin t$$

$$\cos t = \sin t$$

$$t = 45^\circ$$

$$= \frac{1}{10} (\cos 45 + \sin 45) = \frac{\sqrt{2}}{10}$$

$$A \cos t = B \sin \omega t = k \cos(\omega t - 0)$$

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$$A \cos t \neq B \sin wt = k \cos (wt - 0)$$

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But ; $\cos(\omega t - \theta) = \sin(\omega t - \theta + 90^\circ)$.

where $k = \sqrt{A^2 + B^2}$
 $= \sqrt{1/10^2 + 1/10^2} = \sqrt{2}/10$

$\theta = 0^\circ$.

Recall $x = k \sin(t + \alpha)$.

$\frac{\sqrt{2}}{10} = \frac{\sqrt{2}}{10} \sin(45 + \alpha)$

$\alpha = 90 - 45 = 45^\circ$.

Steady state equation

$x = \frac{\sqrt{2}}{10} (\sin t + x/4)$