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A

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

C.f

$$m^2 + 5m + 6 = 0$$

$$m^2 + 2m + 3m + 6 = 0$$

$$m(m+2) + 3(m+2) = 0$$

$$(m+3)(m+2) = 0$$

$$m_1 = -3$$

$$m_2 = -2$$

$$x = Ae^{-3t} + Be^{-2t}$$

P.I

$$f(t) = \cos t$$

$$x = C\cos t + D\sin t$$

$$dx/dt = -C\sin t + D\cos t$$

$$d^2x/dt^2 = -C\cos t - D\sin t$$

$$(-C\cos t - D\sin t) + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t$$

$$= \cos t$$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

$(+5D+6C-C)\cos t + (-5C-D+6D)\sin t = \cos t + 0$

$5C+5D=1 \quad ---i$

$-5C+5D=0 \quad ---ii$

$10C=1$

$C=\frac{1}{10}$

Substitute C in equation i

$5\left[\frac{1}{10}\right]+5D=1$

$\frac{1}{2}+5D=1$

$5D=1-\frac{1}{2}$

$5D=\frac{1}{2}$

$D=\frac{1}{10}$

$\therefore C=D$

general solution =>

$x=Ae^{-3t}+Be^{-2t}+\frac{\cos t}{10}+\frac{\sin t}{10}$

$x=Ae^{-3t}+Be^{-2t}+\frac{1}{10}(\cos t+\sin t)$

when $t=0$ and $x=0.1$

$0.1 = A + B + \frac{1}{10}(1+0)$

$A + B = 0.1 - 0.1$

$A + B = 0 \quad -i$

when $dx/dt = 0$ and $t = 0$

$0 = -3A - 3B + 0.1$

$3A + 3B = 0.1 \quad -ii$

$dx/dt = 3Ae^{-3t} - 2Be^{-2t} + \frac{1}{10}(-\sin t + \cos t)$

from eqn i

$A = -B$

Substituting A in eqn ii

$3(-B) + 3B = 0.1$

$-3B + 3B = 0.1$

$-B = 0.1$

$B = -0.1$

$A = -(-0.1)$

$A = 0.1$

$\therefore x = 0.1e^{-3t} - 0.1e^{-2t} + \frac{1}{10}(\cos t + \sin t)$

$x = 0.1(e^{-3t} - e^{-2t} + (\cos t + \sin t))$

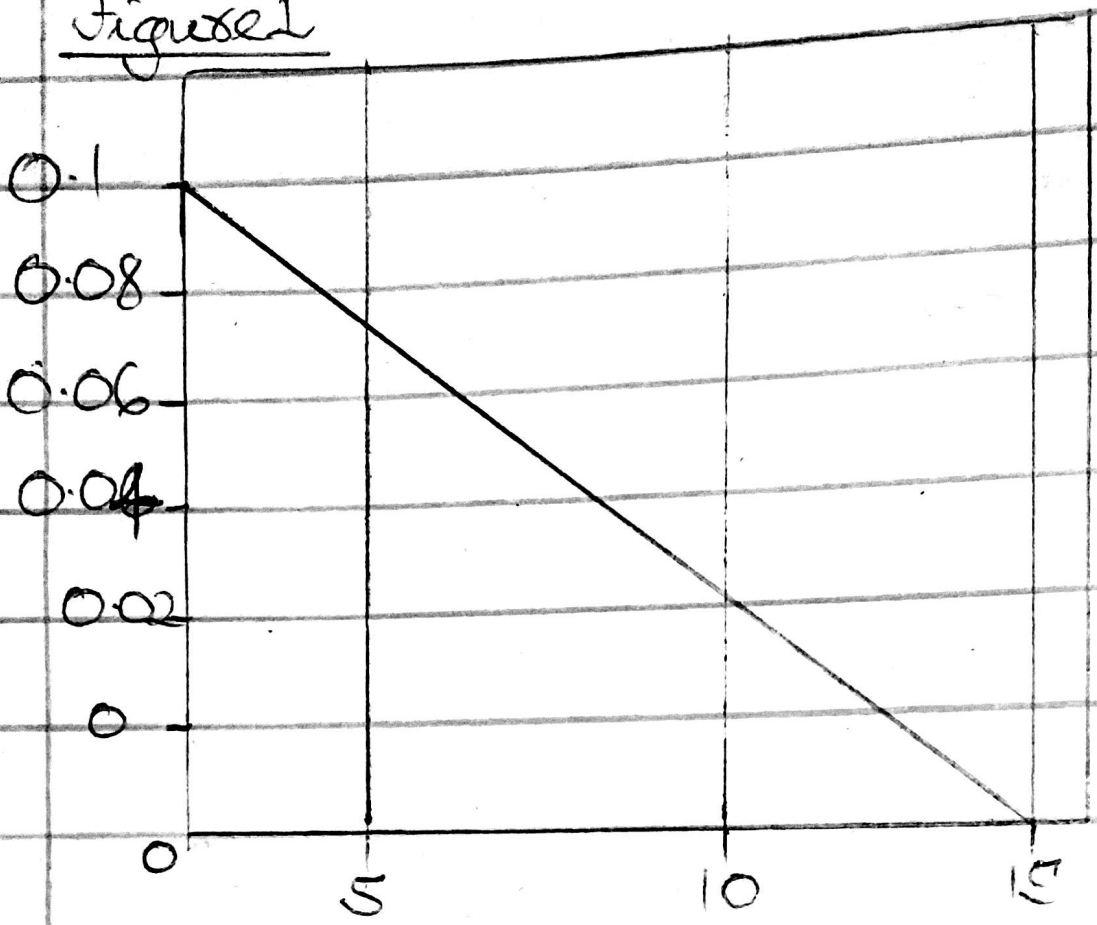
$x = \frac{1}{10}(e^{-3t} - e^{-2t} + (\cos t + \sin t))$

B

Matlab Codes

- Command window
- Clear
- Clc
- Close all
- Symsnt
- x = 0.1*(exp(-3*t) - exp(-2*t) + cos(t) + sin(t))
- tn = [0; 0.01; 15]
- xn = subs(x, tn)
- figure(1)
- plot(tn, xn)
- grid on
- grid ~~number~~ minor
- axis tight
- xlabel('t')
- ylabel('x')

Figure 1



iii

Steady state form

$$x = \frac{1}{10}(e^{-3t} - e^{-2t} + \cos t + \sin t)$$

at steady state $\frac{dx}{dt} = 0$

use considering the steady state part of the equation

$$\frac{dx}{dt} = \frac{1}{10}(-\sin t + \cos t)$$

$$0 = \cos t - \sin t$$

$$\sin t = \cos t$$

$$t = 45^\circ$$

$$x = \frac{1}{x}(\cos 45^\circ + \sin 45^\circ)$$

Recall $A\cos \omega t + B\sin \omega t = \cos(\omega t - \theta)$

$$\cos(\omega t - \theta) = \sin(\omega t - \theta + 90^\circ)$$

$$k = \sqrt{A^2 + B^2} = \sqrt{\left[\frac{1}{10}\right]^2 + \left[\frac{1}{10}\right]^2} = \sqrt{\frac{2}{100}}$$

$$k = \frac{\sqrt{2}}{10}$$

Recall $x = k\sin(t + a)$

$$\frac{\sqrt{2}}{10} = \frac{\sqrt{2}}{10}\sin(45^\circ + a)$$

$$1 = \sin(45 + a)$$

$$45 + a = \sin^{-1}(1)$$

$45 + a = 90$

$a = 45^\circ$

$a = \pi/4$

$x = \frac{\sqrt{2}}{10} \sin(45^\circ + \pi/4)$