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16/ENG02/044
COMPUTER ENGINEERING
ENG 381

Solution to Assignment Questions

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

$m^2 + 5m + 6 = 0$

$m^2 + 3m + 2m + 6 = 0$

$m(m+3) + 2(m+3) = 0$

$(m+2)(m+3)$

$m_1 = -2, \quad m_2 = -3$

C.F $\Rightarrow x = Ae^{-2t} + Be^{-3t}$

P.I $\Rightarrow x = C\cos t + D\sin t$

$$\frac{dx}{dt} = -C\sin t + D\cos t$$

$$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$$

$\Rightarrow -C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t - D\sin t) = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

$-5\cos t + 5D\cos t + 5D\sin t - 5C\sin t = \cos t$

$5C\cos t + 5D\cos t - 5C\sin t + 5D\sin t = \cos t$

$\cos t(5C + 5D) - \sin t(5C - 5D) = \cos t$

comparing coefficients

$5C + 5D = 1$ ——— (i)

$-5C - 5D = 0$ ——— (ii)

$10D = 1$

$\therefore D = \frac{1}{10}$

Sub $D = \frac{1}{10}$ into (i)

$$5C + 5\left(\frac{1}{10}\right) = 1$$

$$5C + \frac{1}{2} = 1$$

$$5C = \frac{1}{2}$$

$$\therefore C = \frac{1}{10}$$

$$\Rightarrow P.I = \frac{1}{10}\cdot\cos t + \frac{1}{10}\sin t$$

$$G.S = C.F + P.I$$

$$\therefore G.S \Rightarrow x = Ae^{-2t} + Be^{-3t} + \frac{1}{10}\left(\cos t + \sin t\right)$$

when $t = 0$, $x = 0.1$ and $\frac{dx}{dt} = 0$

$$\therefore x = Ae^{-2t} + Be^{-3t} + \frac{1}{10}\left(\cos t + \sin t\right)$$

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10}\left(\cos(0) + \sin(0)\right)$$

$$\cancel{\frac{1}{10}} = A + B + \cancel{\frac{1}{10}}$$

$$\Rightarrow A + B = 0$$

Differentiating the General Solution

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - \frac{1}{10}\left(-\sin t + \cos t\right)$$

when $t = 0$, $\frac{dx}{dt} = 0$

$\Rightarrow 0 = -2Ae^{-2(0)} - 3Be^{-3(0)} - \frac{1}{10}\left(-\sin 2(0) + \cos(0)\right)$

$0 = -2A - 3B - \frac{1}{10}$

$\Rightarrow 2A + 3B = \frac{-1}{10}$

$A + \frac{3B}{2} = \frac{-1}{20}$

$\Rightarrow A + B = 0$ —— (i)

$- \quad A + 3/2\,B = \frac{-1}{20}$ —— (ii)

$B - \frac{3}{2}B = \frac{1}{20}$

$\frac{-B}{2} = \frac{1}{20}$

$B = -1/10$

sub $B = {-1}/10$ into eqn (i)

$A - \frac{1}{10} = 0$

$A = +\frac{1}{10}$

$\therefore$ P.S $\Rightarrow x = +\frac{1}{10}e^{-2t} - \frac{1}{10}e^{-3t} + \frac{1}{10}\left(\cos t + \sin t\right)$

or

$x = \frac{1}{10}\left[e^{-2t} - e^{-3t} + \cos t + \sin t\right]$

(iii) $x = K\sin(t + \alpha)$

To solve the steady-state solution

$$x = \frac{1}{10}\left(\left(e^{-2t} + e^{-3t}\right) + \left(\cos t + \sin t\right)\right)$$

Transient part of equation — Steady state part of the equation

Using the steady state part of equation

$x = \frac{1}{10}(\cos t + \sin t)$

Note: $dx/dt = 0$ for steady state

$$\frac{dx}{dt} = \frac{1}{10}\left(-\sin t + \cos t\right) = 0$$

$$-\sin t + \cos t = 0$$

$$\cos t = \sin t$$

Hence: $t = 45^\circ$ or $\pi/4$

$$x = \frac{1}{10}\left(\cos 45 + \sin 45\right)$$

Sinusoidal Equation: $A\cos\omega t + B\sin\omega t = k\cos(\omega t - \theta)$

but $\cos(\omega t - \theta) = \sin(\omega t - \theta + 90^\circ)$

where $K = \sqrt{A^2 + B^2} = \sqrt{(1/10)^2 + (1/10)^2} = 0.14$

Since it is in the same phase, hence $\theta = 0^\circ$

$$\therefore x = \frac{1}{10}\left(\cos 45 + \sin 45\right)$$

$$x = 0.14\sin(45 + 90^\circ)$$

$$x = 0.14\sin(45 + 90)$$

$$x = 0.14\sin(90 + 45)$$

In terms of the above equation, Hence

$K = 0.14$ or $\sqrt{2}/10$

$\alpha = 45^\circ$ or $\pi/2$

$$x = 0.14\sin(t + 45^\circ)$$