$\sin t\,[-B + 5A + 6B] + \cos t\,[-A - 5A + 6B] = \cos t$

$\sin t\,(-5A + 5B) + \cos t\,(5B + 5A) = \cos t$

Equate the coefficients

$-5A + 5B = 0$ — (i)

$5B + 5A = 1$ — (ii)

$-10A = -1$

$A = -1/-10$

$A = 1/10$

$-5A + 5B = 0$

$-5(1/10) + 5B = 0 \quad 5B = 5A$

$5B = 5 \times \frac{1}{10^2}$

$5B = 1/2$

$10B = 1$

$B = \frac{1}{10}$

Particular Integral $\Rightarrow x = 1/10 \cos t + 1/10 \sin t$

$x = \frac{1}{10}[\cos t + \sin t]$

Recall; General solution $\rightarrow x = Ae^{-2t} + Be^{-5t}$

Substituting x and t,

$0.1 = Ae^{-2(0)} + Be^{-5(0)}$

$0.1 = A + B$ — (i)

Substituting $\frac{dx}{dt}$ & $t = 0$

$0 = -3Ae^{-2(0)} - 2Be^{-5(0)}$

$0 = -3A - 2B$ — (ii)

Solve (i) & (ii) simultaneously

$0.1 = A + B$ — (i)

$0 = -3A - 2B$ — (ii)

from (i), $A = 0.1 - B$ — (iii)

$0 = -3(0.1 - B) - 2B$

$0 = -0.3 + 3B - 2B$

$0.3 = 3B - 2B$

$B = 0.3$

from eqn (iii) $A = 0.1 - B$

$A = 0.1 - 0.3$

$A = 0.2$

OKEREKE CHUKWUMA ANTHONY
16/ENG09/092
ELECTRICAL / ELECTRONICS

The dynamic model of a body in motion performing damped forced vibrations is as in Equation (1)

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Given that when $t=0$, $x=0.1$ and $dx/dt = 0$

Solution

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

let $x = Ae^{kt}$

$dx/dt = x' = kAe^{kt}$

$= kx$

$d^2x/dt^2 = x'' = k^2Ae^{kt}$

$= k^2x$

$k^2x + 5kx + 6x$

$x(k^2 + 5k + 6) = 0$

$x=0,\ k^2+5k+6 = 0$

$k^2+5k+6 = 0$ ⇒ The Auxiliary Equation

$k^2 + 3k + 2k + 6 = 0$

$k(k+3) + 2(k+3) = 0$

$(k+2)(k+3) = 0$

$k+2 = 0$ or $k+3 = 0$

$k = -2$ or $k = -3$ ⇒ The Roots of the Equation

$y = Ae^{kx} + Be^{kx}$

$x = Ae^{-2t} + Be^{-3t}$ ⇒ The General Solution

$x = A\cos t + B\sin t$

$dx/dt = -A\sin t + B\cos t$

$d^2x/dt^2 = -A\cos t - B\sin t$

$x = A\cos t + B\sin t$

$dx/dt = -A\sin t + B\cos t$

$d^2x/dt^2 = -A\cos t - B\sin t$

$(-A\cos t - B\sin t) + 5(-A\sin t + B\cos t) + 6(A\cos t + B\sin t) = \cos t$

$-A\cos t - B\sin t - 5A\sin t + 5B\cos t + 6A\cos t + 6B\sin t = \cos t$

$5A\cos t + 5B\cos t - A\sin t - 5A\sin t + 6B\sin t = \cos t$

$5A\cos t + 5B\cos t - 6A\sin t + 6B\sin t = \cos t$

$5A + 5B = 1$

$-6A + 6B = 0$

$-6A = -6B$

$A = B$

$5A + 5A = 1$

$10A = 1$

$A = 0.1$

$B = 0.1$

$x = 0.1\cos t + 0.1\sin t$

General solution → $x = -0.2e^{-2t} + 0.3e^{-3t}$

Complete general solution

→ $x = -0.2e^{-2t} + 0.3e^{-3t} + \frac{1}{10}[\cos t + \sin t]$

Command window

```
clc
syms t
x = -0.2 * exp(-3 * t) + 0.3 * exp(-2 * t) + 0.1 * (cos(t) + sin(t))
tn = [0 : 0.1 : 15]
xn = subs(x, tn)
figure(1)
plot(tn, xn)
xlabel('distance')
ylabel('time')
axis tight
grid on
grid minor
```

Steady state Equation

$x = K\sin(t + \alpha)$

$x = 0.1e^{-2t} + 0.1e^{-3t} + 0.1\{\sin t + \cos t\}$

at steady state

i.e $t \to \infty$

$0.1e^{-2t} \to 0$ and $0.1e^{-3t} \to 0$ at steady state

$x = 0.1\{\sin t + \cos t\}$

$= 0.1\sin t + 0.1\cos t$

$x = K\sin(t + \alpha)$

$\Rightarrow K\sin t\cos\alpha + K\sin\alpha$

Comparing (i) and (ii)

$K\sin\alpha = 0.1$ — (iii)

$K\cos\alpha = 0.1$ — (iv)

$K^2\sin^2\alpha + K^2\cos^2\alpha = 0.1^2 + 0.1^2$

$K^2(\sin^2\alpha + \cos^2\alpha) = 0.02$ → $\sin^2\alpha + \cos^2\alpha = 1$

$K^2 = 0.02$

$K = 0.1414$

$K \sin\alpha = K\cos\alpha = e \cdot 1$

$K \sin\alpha = K\cos\alpha$

$\sin\alpha = \cos\alpha$

$\frac{\sin\alpha}{\cos\alpha} = 1$

$\tan\alpha = 1$

$\alpha = \tan^{-1}(1)$

$\alpha = 45^\circ$

$x = e^{-1}\{\sin t + \cos t\}$

$x = 0.1414 \sin(t + 45^\circ)$