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Matric No:16/MHS03/030

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Solution

The dynamic model of a body in motion performing damped forced vibrations is as in Equation 0

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

$$y = C.F + P.I$$

Find C.F

$m^2 + 5m + 6 = 0$ Auxiliary eqn.

$$m = \frac{-5 \pm \sqrt{25 - 4(6)}}{2}$$

$$= \frac{-5 \pm \sqrt{1}}{2}$$

$m = -2$ or -3

C.F

$$y = Ae^{-2t} + Be^{-3t}$$

P.I

$$x = C\cos t + D\sin t$$

$$\frac{dx}{dt} = -C\sin t + B\cos t$$

$$\frac{d^2x}{dt^2} = -C\cos t - B\sin t$$

Substituting into the initial equation.

$[-C\cos t - D\sin t] + 5[-C\sin t + D\cos t] + 6[C\cos t + D\sin t] = \cos t$

$\therefore -C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

Collect like terms

$-C\cos t + 5D\cos t + 6C\cos t - D\sin t - 5C\sin t + 6D\sin t = \cos t$

$[-C + 5D + 6C]\cos t - [D + 5C - 6D]\sin t = \cos t$

$[5C + 5D]\cos t - [-5D + 5C]\sin t = \cos t$

For $\cos t$

$5C + 5D = 0$ ---- (1)

For $\sin t$

$-5D + 5C = 1$ ---- (2)

From eqn (1)

$\therefore 5C = -5D$

$\therefore C = -\frac{5D}{5} \Rightarrow C = -D$

Substitute into eqn (2) we have

$-5D + 5(-D) = 1$

$\therefore D = 1/10$

$\therefore C = 1/10$

$\therefore$ The general formular is

$x = \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$\therefore$ The complimentary function is

$x = Ae^{-2t} + Be^{-3t} + 1/10\cos t + 1/10\sin t$ --- (3)

With conditioned being

$t = 0$ $x = 0.1$ and $dx/dt = 0$

$\therefore \frac{dx}{dt} = -2Ae^{-2t} - 3e^{-3t} - \frac{1}{10}\sin t + \frac{1}{10}\cos t$ --- (4)

$\therefore$ From equation 3

$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10}\cos(0) + \frac{1}{10}\sin(0)$

$0.1 = A + B + \frac{1}{10}$

$\therefore A + B = 0$ ...... eqn (5)

From eqn 4

$\therefore 0 = -2Ae^{-2(0)} - 3Be^{-3(0)} - \frac{1}{10}\sin(0) + \frac{1}{10}\cos(0)$

$0 = -2A - 3B + \frac{1}{10}$

$\therefore 2A + 3B = 1/10$ .... eqn 6

Comparing eqn 5 and 6 we have

$A = -1/10$ and $B = 1/10$

$\therefore$ The complete solution will be

$x = -\frac{1}{10}e^{-3t} + \frac{1}{10}e^{-3t} - \frac{1}{10}\cos t + \frac{1}{10}\sin t.$