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MECHANICAL ENGINEERING
ENG 381

1) The dynamic model of a body in motion performing damped forced vibrations is as in Eqn (1)

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t \quad \text{——————(1)}$$

Given that when $t = 0$, $x = 0.1$ and $\frac{dx}{dt} = 0$,

+ ~~using the Auxiliary Equation~~

(i) using the Auxiliary Eqn method, obtain the solution of the model in form of an expression having x as a function of t,

(ii) Write a MATLAB program to plot the relationship between x and t for $0 \leq t \leq 15$ unit using a step size of 0.01 unit and

(iii) Write the steady-state solution of the system in form $x = k\sin(t+a)$

Soln

1 $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

Cf; $m^2 + 5m + 6 = 0$; $m^2 + 2m + 3m + 6 = 0$

$m(m+2) + 3(m+2) = 0$

~~not~~ $(m+3)(m+2) = 0$

$m_1 = -3$, $m_2 = -2$

$\therefore x = Ae^{-3t} + Be^{-2t}$,

PI: $f(t) = \cos t$

$x = C\cos t + D\sin t$

$\frac{dx}{dt} = -[C\sin t + D\cos t]$

$\frac{d^2x}{dt^2} = -[C\cos t - D\sin t]$

$(-C\cos t - D\sin t) + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

$(5D - C + 6C)\cos t + (-5C - D + 6D)\sin t = \cos t + 0$

Relating LHS & RHS

$5C + 5D = 1 \quad \text{---(1)}$

$-5C + 5D = 0 \quad \text{---(2)}$

$5D = 5C$

$D = C$

Substituting for D in (1)

$5C + 5C = 1$

$10C = 1$

$C = 1/10 = D$

PI $\Rightarrow x = \frac{\cos t}{10} + \frac{\sin t}{10}$

General Soln $\Rightarrow$

$x = Ae^{-3t} + Be^{-2t} + \frac{\cos t}{10} + \frac{\sin t}{10}$

$x = Ae^{-3t} + Be^{-2t} + \frac{1}{10}(\cos t + \sin t)$

When $t = 0$ & $x = 0.1$

$0.1 = A + B + \frac{1}{10}(1 + 0)$

$A + B = 0 \quad \text{---(1)}$

$$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} + 0.1(-\sin t + \cos t)$$

When $dx/dt = 0$; $t = 0$

$0 = -3A - 2B + 0.1(-0+1)$

$3A + 2B = 0.1$ ——— (2)

from eqn A

$A = -B$

Sub for A in eqn (2)

$3(-B) + 2B = 0.1$

$-1B = 0.1$

$B = -0.1$

$A = 1B$

$A = 0.1$

$x = 0.1 \times e^{-3t} - 0.1 \times e^{-2t} + 0.1(\cos t + \sin t)$

$x = 0.1(e^{-3t} - e^{-2t} + \cos t + \sin t)$

$x = \frac{1}{10}.(e^{-3t} - e^{-2t} + \cos t + \sin t)$.

(ii) MatchaB codes

```
0 - Command window
2 - Clear
3 - Clc
4 - Close all
5 - Syms t
6 - x = 0.1 x (exp(-3xt) - exp(-2xt) + cos(t) + sin(t)
7 - tn = [0; 0.01; 15]
8 - xn = subs (x, tn)
9 - figure (1)
10 - plot (tn, xn)
11 - grid on
12 - grid mirror
```

13 - axis tight
14 - xlabel ('time')
15 - ylabel ('vibrations')
16 - axis tight

iii) $x = \frac{1}{10}(e^{-3t} - e^{-3t} + \sin t + \cos t)$

when at steady state $\frac{dx}{dt} = 0$ i.e

Change in x with time is zero

$\therefore \frac{dx}{dt} = \frac{1}{10}(-3e^{-3t} - e^{-3t} + \cos t - \sin t)$

$0 = \cos t - \sin t$

$\cos t = \sin t$

$t = 45°$

$\therefore x = 1/10(\cos 45 + \sin 45) = \sqrt{2}/10$

$A\cos\omega t + B\sin\omega t = k\cos(\omega t - \theta)$

But; $\cos(\omega t - \theta) = \sin(\omega t - \theta + 90°)$

where $k = \sqrt{A^2 + B^2} = \sqrt{(1/10)^2 + (1/10)^2} = \sqrt{1+1}/(10)^2$

$k = \sqrt{2}/10$ //

$\theta = 0°$ (since A & B in same phase

Recall that $x = k\sin(t + a)$

$\sqrt{2}/10 = \sqrt{2}/10 \sin(45 + a)$, $1 = \sin(45 + a)$

$45 + a = \sin^{-1}(1)$

$a = 90 - 45 = 45° = \pi/4$

The steady state solution = $x = \frac{\sqrt{2}}{10}\sin(t + \pi/4)$ //