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16/ENG 061 010

ENG 381

$$1) \frac{d^2 x}{dt^2} + 5 \frac{dx}{dt} + 6x = \cos t$$

$$C.F = m^2 + 5m + 6$$

$$m^2 + 3m + 2m + 6 = 0$$

$$m(m+3) + 2(m+3)$$

$$(m+3)(m+2)$$

$$m = -3 \text{ or } m = -2$$

$$x = Ae^{-3t} + Be^{-2t}$$

$$P.I = x_p = \cos t$$

$$x = A \cos t + B \sin t$$

$$\frac{dx}{dt} = -A \sin t + B \cos t$$

$$\frac{d^2 x}{dt^2} = -A \cos t - B \sin t$$

$$(A \cos t - B \sin t) + 5(-A \sin t + B \cos t) + 6(A \cos t + B \sin t) = \cos t$$

$$\cos t (5B + 5A) + \sin t (6B - 5A) = 5B + 5A = 1$$

$$+ 5B - 5A = 0$$

$$10B = 1$$

$$B = \frac{1}{10}$$

$$5\left(\frac{1}{10}\right) + 5A = 1$$

$$\frac{1}{2} + 5A = 1$$

$$5A = 1 - \frac{1}{2}$$

$$A = \frac{1}{2} \div 5$$

$$A = \frac{1}{10}$$

$$P.I = \frac{1}{10} \cos t + \frac{1}{2} \sin t$$

$$x = C.E + P.I$$

$$x = Ae^{-3t} + Be^{-2t} + \frac{1}{10} (\cos t + \sin t)$$

$$\text{When } x = 0.1 \text{ at } t = 0$$

$$0.1 = Ae^0 + Be^0 + \frac{1}{10} (\cos 0 + \sin 0)$$

$$0 = A + B + \frac{1}{10}$$

$$A + B = \frac{1}{10} + 0.1$$

$$A + B = 0$$

$$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} + \frac{1}{10} (-\sin t + \cos t)$$

When $dx/dt = 0 \quad t = 0$

$$0 = -3Ae^0 - 2Be^0 + \frac{1}{10} (\sin(0) + \cos(0))$$

$$0 = -3A - 2B + \frac{1}{10} (0+1)$$

$$0 = -3A - 2B + \frac{1}{10}$$

$$3A + 2B = \frac{1}{10}$$

$$\begin{cases} \therefore 3A + 2B = \frac{1}{10} & \times 1 \\ A + B = 0 & \times 2 \end{cases}$$

$$3A + 2B = \frac{1}{10}$$

$$2A + 2B = 0$$

$$A = \frac{1}{10}$$

$$3 \left(\frac{1}{10} \right) + 2B = \frac{1}{10}$$

$$\frac{3}{10} + 2B = \frac{1}{10}$$

$$2B = \frac{1}{10} - \frac{3}{10}$$

$$B = \frac{2}{10} \div 2$$

$$B = \frac{1}{10}$$

Q2, math lab

3, from the graph A amplitude is 0.141 ; $A \sin(\omega t + \phi)$

$$T = 14.9 - 8.7 = 6.2 \text{ sec}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{7.2} = 1.0 \text{ rad/sec or } \frac{\pi}{2} \text{ rad}$$

for the value of t for which $\sin t = \cos t$
 $\Rightarrow 45^\circ$ or $\frac{\pi}{4} \text{ rad}$

The steady state solution is given as

$$0.141 \sin(t + 45^\circ)$$

OR

$$\frac{\sqrt{2}}{10} \sin(t + \frac{\pi}{4})$$

COPMMANDS

Commandwindow

Clear

Clc

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syms t

x = 0.1*(exp(-3*t))-exp(-

2*t)+cos(t)+sin(t)

tn = [0;0.01;15] xn = subs(x,tn)

figure (1) plot(tn,xn) grid on grid minor axis tight xlabel ('t') ylabel ('x')

