i.e $x = 1/10\,(\cos t + \sin t)$

$x = 0.1\cos t + 0.1\sin t$

Also; $x = K\sin(t + a)$

From trig. $\rightarrow K\sin t + \cos a + K\sin a \cos t$

$\therefore K\sin t \cos a + K\sin a \cos t = 0.1\cos t + 0.1\sin t$

Taking coefficients of $\sin t$

$K\cos a = 0.1$

Taking coefficient of $\cos t$

$K\sin a = 0.1$

Recall that $K^2 = A^2 + B^2$

$K^2\cos^2 a + K^2\sin^2 a = 0.1^2 + 0.1^2$

$K^2(\cos^2 a + \sin^2 a) = 0.1^2 + 0.1^2$

$K^2 \cdot (1) = 0.1^2 + 0.1^2$

$K = \sqrt{0.1^2 + 0.1^2}$

$K = \sqrt{2}/10$ and

$K\cos a = K\sin a$

$\cos a = \sin a$

$\dfrac{\sin a}{\cos a} = 1$

$\tan a = 1$

$a = \tan^{-1}(1)$

$a = 45°$

Therefore the steady state eqn will be;

$$x = \frac{\sqrt{2}}{10}\sin(t + 45°)$$

$2A + 3B = 0.1$

$2A + 2B = 0$

$B = 0.1$ and multiply eqn 2 by 3

$2A + 3B = 0.1$

$3A + 8B = 0$

$-A = 0.1$

$A = -0.1$

Therefore, the particular solution will be

$x = -\frac{1}{10}e^{-2t} + \frac{1}{10}e^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$x = \frac{1}{10}[-e^{-2t} + e^{-3t} + \cos t + \sin t]$

For the steady state equation

$\frac{dx}{dt} = 0$

$\therefore \frac{dx}{dt} = \frac{1}{10}[2e^{-2t} - 3e^{-3t} - \sin t + \cos t$

$0 = \frac{1}{10}[2e^{-2t} - 3e^{-3t} - \sin t + \cos t]$

$0 = 2e^{-2t} - 3e^{-3t} - \sin t + \cos t$

$\sin t - \cos t = 2e^{-2t} - 3e^{-3t}$

where the relationship between trigonometric function and exponential function is given below:

$\sin t = \frac{1}{2m}[e^{+2t} - e^{-3t}]$

$\cos t = \frac{1}{2m}[e^{+2t} - e^{-3t}]$

$\therefore \sin t - \cos t = \frac{1}{2m}[e^{2t} - e^{3t}] - \frac{1}{2m}[e^{2t} + e^{-3t}]$

$\sin t - \cos t = 0$

$\sin t = \cos t$

Dividing through by $\cos t$

$\frac{\sin t}{\cos t} = \frac{\cos t}{\cos t}$

$\tan t = 1$

$t = \tan^{-1}$

$t = 45°$

Since the graph is sinusoidal, the exponentials will be zero

Solving both equations simultaneously

$5D + 5C = 1$

$-5C + 5D = 0$

Solving both equations simultaneously

$5D + 5C = 1$

$\underline{5C + 5D = 0}$

$5D + 5C = 1$

$\underline{5D - 5C = 0}$

$10D = 1$

$D = 1/10$ and

$10C = 1$

$C = 1/10$

Therefore, the general solution will be

$x = Ae^{-2t} + Be^{-3t} + x_1$

where $x_1 = 1/10 \cos t + 1/10 \sin t$

$\therefore x = Ae^{-2t} + Be^{-3t} + 1/10 \cos t + 1/10 \sin t$

For conditions

$x = 0.1$ and $t = 0$

$0.1 = Ae^{0} + Be^{0} + 1/10 \cos(0) + 1/10 \sin(0)$

$0.1 = A + B + 1/10$

$0.1 - 1/10 = A + B$

$A + B = 0$ —— ①

For condition

$t = 0.1$ and $dx/dt = 0$

$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - 1/10 \sin t + 1/10 \cos t$

$0 = -2Ae^{0} - 3Be^{0} - 1/10 \sin(0) + 1/10 \cos(0)$

$0 = -2A - 3B + 1/10$

$2A + 3B = 0.1$ —— ②

Treating both equations simultaneously

$2A + 3B = 0.1$ —— $\times 1$

$A + B = 0$ —— $\times 2$

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$$\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t$$

Given that when $t = 0$, $x = 0.1$ and $\frac{dx}{dt} = 0$

Using the auxillary method

$m^2 + 5m + 6 = 0$

$m^2 + 3m + 2m + 6 = 0$

$(m^3 + 3m) + (2m + 6) = 0$

$m(m+3) + 2(m+3) = 0$

$(m+2)(m+3) = 0$

$\therefore m = -2$ and -3

Roots are real and different

Therefore, the general solution will be:

$x = Ae^{m_1 t} + Be^{m_2 t}$

$x = Ae^{-2t} + Be^{-3t}$

Finding the particular integral

$F(x) = \cos t$

$x = C\cos t + D\sin t$

$\frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

Subbing into the original equation

$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

$-C\cos t + 5D\cos t + 6C\cos t - D\sin t - 5C\sin t + 6D\sin t = \cos t$

$\cos t(-C + 5D + 6C) + \sin t(-D - 5C + 6D) = \cos t$

$\cos t(5D + 5C) + \sin t(-5C + 5D) = \cos t$

Take the coefficients of $\cos t$

$5D + 5C = 1$

Take the coefficents of $\sin t$

$-5c + 50 = 0$