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Linear Solution

16/Eng03/032

Mechanics Engineering  
Engineering maths II

$$\frac{d^2 x}{dt^2} + 5 \frac{dx}{dt} + 6x = \cos t \quad \dots \textcircled{1}$$

when  $t=0$ ,  $x=0.1$  and  $\frac{dx}{dt}=0$

$$\text{Let } x = e^{kt}$$

$$\frac{dx}{dt} = ke^{kt} = kx$$

$$\frac{d^2 x}{dt^2} = k^2 e^{kt} = k^2 x$$

$$k^2 x + 5kx + 6x = 0$$

$$k^2 + 5k + 6 = 0$$

$$k^2 + 5k + 6 = 0$$

$$k^2 + 2k + 3k + 6 = 0$$

$$k(k+2) + 3(k+2) = 0$$

$$(k+2)(k+3) = 0$$

$$k_1 = -2 \quad k_2 = -3$$

$$y = A e^{k_1 t} + B e^{k_2 t}$$

$$y = A e^{-2t} + B e^{-3t}$$

General form of RHS

$$= (\cos t + A \sin t)$$

$$\frac{dx}{dt} = -(\sin t + A \cos t)$$

$$\frac{d^2 x}{dt^2} = -(\cos t - A \sin t)$$

Substitute back in equation ①

$$-(\cos t - A \sin t) + 5(-\sin t + A \cos t) + 6(\cos t + A \sin t)$$

$$= \cos t$$

$$-(\cos t - A \sin t - 5(\sin t + 5A \cos t) + 6(\cos t + 6A \sin t)$$

$$= \cos t \quad 5(\cos t + 5A \cos t + 5A \sin t - 5 \sin t) = \cos t$$

$$(5c + 5A) \cos t + (5A - 5c) \sin t = \cos t$$

$$5c + 5A = 1 \quad \text{--- ①}$$

$$-5c + 5A = 0 \quad \text{--- ②}$$

$$-10c = 1, \quad c = 1/10$$

from eq. (2)

$$-5C + 9D = 0$$

$$-6\left(\frac{1}{10}\right) + 5D = 0$$

$$\frac{1}{2} + 5D = 0 \quad 5D = -\frac{1}{2} \quad D = -\frac{1}{10}$$

PI =  $\frac{1}{10} \cos t + \frac{1}{10} \sin t$

Complete general solution

$$x = Ae^{-2t} + Be^{3t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

at  $t=0, x=0.1, \frac{dx}{dt}=0$

$$0.1 = A + B + \frac{1}{10} \quad (i)$$

$$0 = -2A + 3B + \frac{1}{10} \quad (ii)$$

$$0.1 = A + B + \frac{1}{10} \quad (iii)$$

$$-A + B = 0 \quad (iv)$$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{3t} + \frac{1}{10}(-\sin t + \cos t)$$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{3t} + \frac{1}{10}(-\sin t + \cos t)$$

$$0 = -2A - 3B + \frac{1}{10} \quad (v)$$

$$0 = -2A - 3B + \frac{1}{10} \quad (vi)$$

$$2A + 3B = \frac{1}{10} \quad (vii)$$

$$2A + 3B = 0 + 2 \quad (viii)$$

$$A + 3B = 0 + 2$$

$$2A + 3B = \frac{1}{10}$$

$$2A + 2B = 0$$

$$B = \frac{1}{10}$$

from eq. (iii)

$$A + B = 0$$

$$A + \frac{1}{10} = 0$$

$$-A = -\frac{1}{10} \quad A = \frac{1}{10}$$

$$x = \frac{1}{10}e^{-2t} + \frac{1}{10}e^{3t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$x = \frac{1}{10}e^{-2t} + \frac{1}{10}e^{3t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$


