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$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

for the General solution.

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$$

$$k^2 + 5k + 6 = 0$$

$$k^2 + 2k + 3k + 6 = 0$$

$$k(k+2) + 3(k+2)$$

$$(k+3)(k+2) = 0$$

$$k_1 = -3, k_2 = -2$$

$$x = Ae^{-3t} + Be^{-2t} \rightarrow \text{complementary soln.}$$

$$x = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

$$-C \cos t - D \sin t + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t$$

$$\cos t(-C + 5D + 6C) + \sin t(-D - 5C + 6D) = \cos t$$

$$-C + 5D + 6C = 1$$

$$A = -B$$

$$0 = 3B - 2B + 0.1$$

$$0 = B + 0.1$$

$$B = -0.1$$

$$A = 0.1$$

$$x = 0.1e^{-3t} - 0.1e^{-2t} + 0.1 \cos t + 0.1 \sin t$$

The steady state solution of the system is in the form of $x = k \sin(t + a)$

at steady state.

$$0.1e^{3t} \rightarrow 0 \text{ and } 0.1e^{-2t} \rightarrow 0$$

$$x = 0.1 (\cos t + \sin t)$$

$$= 0.1 \cos t + 0.1 \sin t.$$

$$x = k \sin(t + a)$$

$$x = k_1 \sin t \cos a + k \sin a \cos t$$

$$k \sin a = 0.1 \rightarrow 1b$$

$$k \cos a = 0.1 \rightarrow 1c$$

$$k^2 \sin^2 a + k^2 \cos^2 a = 0.1^2 + 0.1^2$$

$$k^2 (\sin^2 a + \cos^2 a) = 0.02$$

$$k^2 = 0.02$$

$$k = 0.141$$

$$k \sin a = k \cos a = 0.1$$

$$\frac{\sin a}{\cos a} = 1$$

$$\tan a = 1$$

$$\tan a = 1$$

$$a = \tan^{-1}(1)$$

$$a = 45^\circ$$

$$x = 0.1 (\cos t + \sin t)$$

$$\rightarrow x = 0.141 \sin(t + 45^\circ)$$

$$5C + 5D = 1 \quad \text{--- (1)}$$

$$-5C + 5D = 0 \quad \text{--- (2)}$$

Adding equation (1) and (2)

$$10D = 1$$

$$D = \frac{1}{10}$$

$$5C = 1 - 5 \cdot \frac{1}{10}$$

$$5C = 1 - \frac{1}{2}$$

$$5C = \frac{1}{2}$$

$$C = \frac{1}{10}$$

$$C = D = \frac{1}{10}$$

$$x = \frac{1}{10} \cos t + \frac{1}{10} \sin t \quad \rightarrow \text{Particular Interest.}$$

$$x = Ae^{-3t} + Be^{-2t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$t = 0, \quad x = 0.1, \quad \frac{dx}{dt} = 0$$

$$x = Ae^{-3t} + Be^{-2t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} - \frac{1}{10} \sin t + \frac{1}{10} \cos t$$

$$0.1 = Ae^{-3(0)} + Be^{-2(0)} + \frac{1}{10} \cos(0) + \frac{1}{10} \sin(0)$$

$$0.1 = A + B + \frac{1}{10} + 0$$

$$0.1 = A + B + 0.1$$

$$A + B = 0, \quad A = -B$$

$$0 = -3Ae^{-3(0)} - 2Be^{-2(0)} - \frac{1}{10} \sin 0 + \frac{1}{10} \cos 0$$

$$0 = -3A - 2B + \frac{1}{10}$$