EDET, UBONG UMOH
16/ENG06/023
MECHANICAL ENGR.
300L
ENG381 (ENGR. MATH III)

① $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

$\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = 0$

$x = Ae^{mt}$ for C.F

$m^2 + 5m + 6 = 0$

$m^2 + 3m + 2m + 6 = 0$

$m(m+3) + 2(m+3) = 0$

$(m+3)(m+2) = 0$

$m_1 = -3$ and $m_2 = -2$

$x = Ae^{m_1 t} + Be^{m_2 t}$

$x = Ae^{-3t} + Be^{-2t}$ —— C.F

for P.I

$u : f(x) = \cos t$

$x = A\cos t + B\sin t$

$\frac{dx}{dt} = -A\sin t + B\cos t$

$\frac{d^2x}{dt^2} = -A\cos t - B\sin t$

$(-A\cos t - B\sin t) + (-5A\sin t + 5B\cos t) + (6A\cos t + 6B\sin t) = \cos t$

$(-A\cos t + 6A\cos t) + (-B\sin t + 6B\sin t) + (-5A\sin t + 5B\cos t) = \cos t$

$5A\cos t + 5B\sin t - 5A\sin t + 5B\cos t = \cos t$

Collecting co-efficients of their like terms.

$\Rightarrow 5A + 5B = 1$ ——— equ (1)

$+(-)\ 5A + 5B = 0$ ——— equ (2)

$10B = 1$

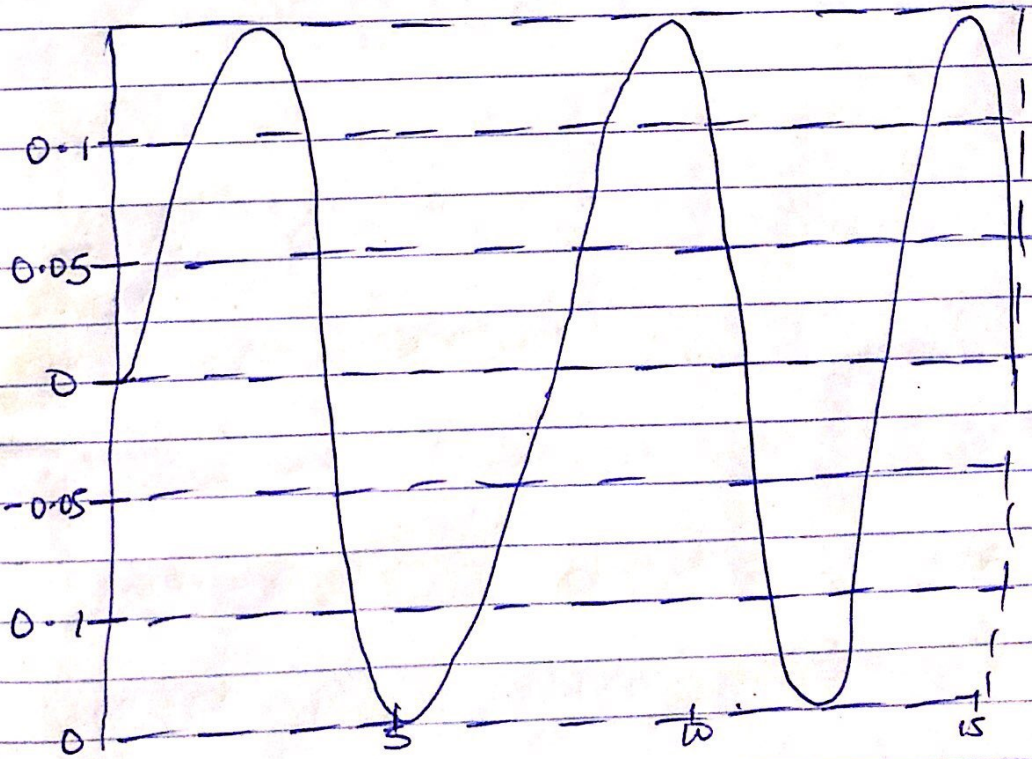
$B = 1/10$

Put $B = 1/10$ in equ (1)

$5A + 5\left(\frac{1}{10^2}\right) = 1$

Commandwindow.

```
clear
clc
close all
Syms t
x = -2*exp(-3*t)/10+exp(-2*t)/10+cos(t)/10+sin(t)/10
tn = (0;0.01;15)
xn = subs(x,tn)
plot(tn,xn)
xlabel = ('time')
ylabel = ('variable')
axis tight
grid on
grid minor.
```



Graph of -2*exp(-3*t).

$5A + \frac{1}{2} = 1$

$5A = 1 - \frac{1}{2} \Rightarrow 5A = \frac{1}{2} \Rightarrow 10A = 1$

$A = \frac{1}{10}$

P.I $\Rightarrow \frac{1}{\omega}\cos t + \frac{1}{10}\sin t$

$P.I = \frac{1}{10}(\cos t + \sin t)$

General Solution.

$x = C.F + P.I$

$x = Ae^{-3t} + Be^{-2t} + \frac{1}{10}(\cos t + \sin t)$

C.F

P.I

(iii) $x = k\sin(t + a)$

solution.

$x = \frac{1}{10}\left[e^{-3t} - e^{-2t} + \sin t + \cos t\right]$

At steady state $\frac{dx}{dt} = 0$.

Change in x with time in zero.

$\therefore \frac{dx}{dt} = \frac{1}{10}(-3e^{-3t} - e^{-2t} + \cos t - \sin t)$

Note; the exponentials result zero.

$0 = \cos t - \sin t$

$\cos t = \sin t$

$t = 45^\circ \Rightarrow x = \frac{1}{10}[\cos 45 + \sin 45]$

from sinosodial expressions,

$A\cos\omega t + B\sin\omega t = k\cos(\omega t - \theta)$.

But; $\cos(\omega t - \theta) = \sin(\omega t - \theta + 90^\circ)$.

where, $k = \sqrt{A^2 + B^2} = \sqrt{(1/10)^2 + (1/10)^2} = \sqrt{\frac{1+1}{10^2}}$

$k = \frac{\sqrt{2}}{10}$ $\theta = 0^\circ$ (since its in same phase)

Recall $x = k\sin(t + a)$ $\frac{\sqrt{2}}{10} = \frac{\sqrt{2}}{10}\sin(45 + a)$