

FLORINA GEMANUEL ENOJO

CIVIL ENGINEERING

161ENG031032

$$\textcircled{1} \frac{d^2 x}{dt^2} + 5 \frac{dx}{dt} + 6x = \cos t$$

$$\text{cf: } m^2 + 5m + 6 = 0 \quad ; \quad m^2 + 2m + 3m + 6 = 0$$

$$m(m+2) + 3(m+2) = 0 \quad ; \quad m_1 = -3 \quad \& \quad m_2 = -2$$

$$\therefore x_c = Ae^{-3t} + Be^{-2t}$$

$$\text{P.I: } f(t) = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2 x}{dt^2} = -C \cos t - D \sin t$$

$$(-C \cos t - D \sin t) + 5(C \cos t - D \sin t) + 6(C \cos t + D \sin t)$$

$$= \cos t$$

$$-(C \cos t - D \sin t - 5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t) = \cos t$$

$$(5D - C + 6C) \cos t + (-5C - D + 6D) \sin t = \cos t + 0$$

Relating L.H.S & R.H.S

$$5C + 5D = 1 \quad \dots \textcircled{1}$$

$$-5C + 3D = 0 \dots (2)$$

$$5D = 5C$$

$$D = C$$

Substituting for D in (1)

$$5C + 5C = 1$$

$$10C = 1$$

$$C = \frac{1}{10} = D$$

$$P.T \Rightarrow x = \frac{\cos t}{10} + \frac{\sin t}{10}$$

General Soln \Rightarrow

$$x = A e^{-3t} + B e^{-2t} + \frac{\cos t}{10} + \frac{\sin t}{10}$$

$$x = A e^{-3t} + B e^{-2t} + \frac{1}{10} (\cos t + \sin t)$$

When $t = 0$ $\frac{1}{3} x = 0.1$

$$0.1 = A + B + \frac{1}{10} (1 + 0)$$

$$A + B = 0 \dots *$$

$$\frac{dx}{dt} = -3A e^{-3t} - 2B e^{-2t} + \frac{1}{10} (-\sin t + \cos t)$$

When $\frac{dx}{dt} = 0$: $t = 0$

$$0 = -3A - 2B + 0.1 (-0 + 1)$$

$$3A + 2B = 0.1 \dots **$$

from eqn A

$$A = -B$$

$$A = 0.1$$

$$\therefore x = 0.1 * e^{-3t} - 0.1 * e^{-2t} + 0.1 (\cos t + \sin t)$$

$$x = 0.1 (e^{-3t} - e^{-2t} + \cos t + \sin t)$$

2

5. mat lab Codes

- Command window

- clear

- clc

- close all

- Syms ~~t~~

- $x = 0.1 * (\exp(-3*t) - \exp(-2*t) + \cos(t) + \sin(t))$

- $t_n = [0; 0.01; 15]$

- $x_n = \text{subs}(x, t_n)$

- figure (1)

- plot (t_n, x_n)

- grid on

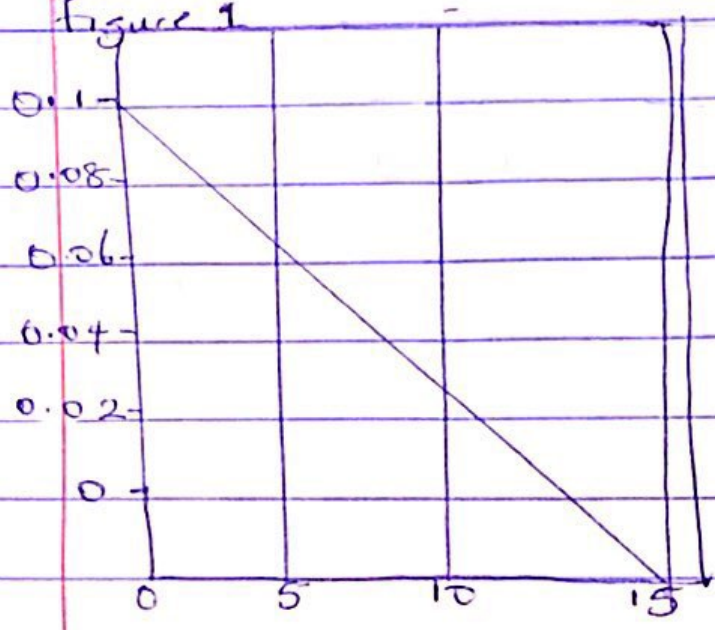
- grid minor

- axis tight

- x label ('t')

- y label ('x')

Figure 1



iii)