Ebitu Paul Nsien
17/ENG06/095
ENG 381 (Engineering mathematics IV)
Mechanical Engineering

(1)

Equation 1

$d^2x/dt^2 + 5dx/dt + 6x = \cos t$

showing the dynamic model of a body in motion performing damped forced vibrations

Given that $t=0$, $x=0.1$ and $dx/dt = 0$

i $d^2x/dt^2 + 5dx/dt + 6x = \cos t$

C.F $= m^2 + 5m + 6 = 0$, $m^2 + 2m + 3m + 6 = 0$

$m(m+2) + 3(m+2) = 0$; $m = -3$ & $m = -2$

$\therefore x = Ae^{-3t} + Be^{-2t}$ //

P.I $= f(t) = \cos t$; $x = C\cos t + D\sin t$

$dx/dt = -C\sin t + D\cos t$

$d^2x/dt^2 = -C\cos t - D\sin t$

$C - (C\cos t - D\sin t) + 5C - C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6\cos t + 6D\sin t = \cos t$

$(5D - C + 6C)\cos t + (-5C - D + 6D)\sin t = \cos t + 0$

L.H.S $\rightarrow$ R.H.S

$5C + 5D = 1$ ... (1)

$-5C + 5D = 0$

$5D = 5C$, $D = C$

$5C + 5C = 1$

$10C = 1$

$C = 1/10 = D$

PI $\Rightarrow x = \cos t/10 + \sin t/10$

$\Rightarrow$

$x = Ae^{-3t} + Be^{-2t} + \cos t/10 + \sin t/10$

$x = Ae^{-3t} + Be^{-2t} + 1/10(\cos t + \sin t)$

when $t = 0$ & $x = 0.1$

$0.1 = A + B + 1/10(1 + 0)$

$A + B = 0$ --- (1)

$dx/dt = -3Ae^{-3t} - 2Be^{-2t} + 1/10(-\sin t + \cos t)$

when $dx/dt = 0$; $t = 0$

$0 = -3A - 2B + 0.1(-0 + 1)$

$3A + 2B = 0.1$ --- (2)

from eq (1)

$A = -B$

Substituting for A in eq. (2)

$3(-B) + 2B = 0.1$

$-B = 0.1$

$B = -0.1$

$A = -B$

$A = 0.1$

$\therefore x = 0.1e^{-3t} - 0.1e^{-2t} + 0.1(\cos t + \sin t)$

$x = 0.1(e^{-3t} - e^{-2t} + \cos t + \sin t)$

$x = 1/10(e^{-3t} - e^{-2t} + \cos t + \sin t)$

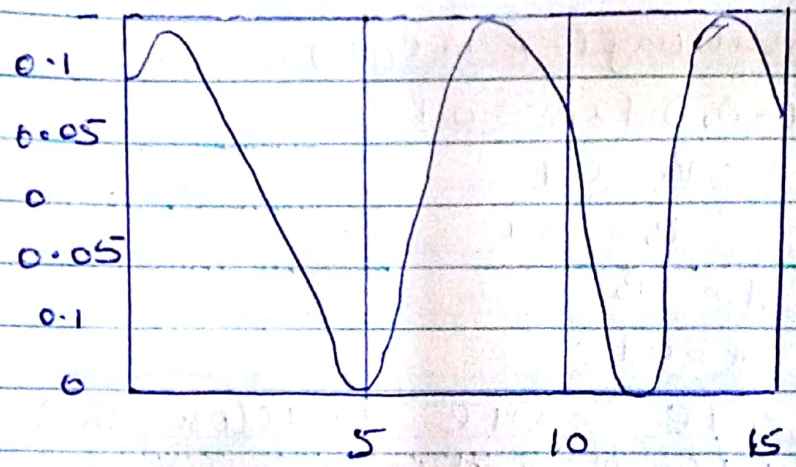
11 Write a Matlab program to plot the relationship between x & t for $0 \le t \le 15$ unit using a step size of 0.01 unit.

Soln

- Commandwindow
- clear
- clc
- close all

- Syms t
- x = 0.1(exp(-3t) - exp(-2t)) + cos(t) + sin(t)
- tn = [0; 0.01; 15]
- xn = subs(x, tn)
- figure (1)
- Plot (tn, xn)
- grid on
- grid minor
- xlabel ('time')
- ylabel ('vibrations')
- axis tight

Figure 1



$\therefore x = A\sin(\omega t + \alpha)$

Soln

$x = 1/10\,(e^{-3t} - e^{-2t} + \sin t + \cos t)$

At steady rate $dx/dt = 0$ i.e

Change in x with time $dx/dt = 0$

$dx/dt = 1/10\,(3e^{-3t} - e^{-2t} + \cos t - \sin t)$

$0 = \cos t - \sin t$

$\cos t = \sin t$

$t = 45°$

$x = 1/10\,(\cos 45 + \sin 45) = \sqrt{2}/10$

from Sinosodial expression

$A\cos\omega t + B\sin\omega t = K\cos(\omega t - \theta)$

But; $\cos(\omega t - \theta) = \sin(\omega t - \theta + 90^\circ)$

where

$K = \sqrt{A^2 + B^2} = \sqrt{(1/10)^2 + (1/10)^2} = \sqrt{1+1}/10^2$

$K = \sqrt{2}/10$ A

$\theta = 0^\circ$

from

$x = K\sin(t + \alpha)$

$\sqrt{2}/10 = \sqrt{2}/10 \sin(45 + \alpha)$

$1 = \sin(45 + \alpha)$

$45 + \alpha = \sin^{-1}(1)$

$\alpha = 90 - 45 = 45^\circ = \pi/4$

$x = \sqrt{2}/10 \sin(t + \pi/4)$

the steady state equation.