$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \text{Cos}t$$

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$$

$x = Ae^{kx}$

$dx/dt = KAe^{kx}$

$d^2x/dt^2 = K^2Ae^{kx}$

$K^2x + 5Kx + 6x = 0$

$K^2 + 5K + 6 = 0$

$$K = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{25-24}}{2}$$

$$= \frac{-5 \pm 1}{2}$$

$= -2$ or -3

$x = Ae^{-2t} + Be^{-3t}$

$x = C\,\text{Cos}t + D\,\text{Sin}t$

$dx/dt = -C\,\text{Sin}t + D\,\text{Cos}t$

$d^2x/dt^2 = -C\,\text{Cos}t - D\,\text{Sin}t$

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \text{Cos}t$$

$[-C\,\text{Cos}t - D\,\text{Sin}t] + 5[-C\,\text{Sin}t + D\,\text{Cos}t] + 6[C\,\text{Cos}t + D\,\text{Sin}t] = \text{Cos}t$

$-C\,\text{Cos}t - D\,\text{Sin}t - 5C\,\text{Sin}t + 5D\,\text{Cos}t + 6C\,\text{Cos}t + 6D\,\text{Sin}t = \text{Cos}t$

$[-C + 5D + 6C]\,\text{Cos}t + [-D - 5C + 6D]\,\text{Sin}t = \text{Cos}t$

$[5D + 5C]\,\text{Cos}t + [-5C + 5D]\,\text{Sin}t = \text{Cos}t$

$\text{Cos}t : [5D + 5C = 1]$

$\text{Sin}t : [5D - 5C = 0]$

$10D = 1$

$D = 1/10 \;:\; 5(1/10) + 5C = 1 \;:$

~~Partiqulat Intergral~~ : $\cancel{x}$ $1/2 + 5C = 1$

$5C = 1 - 1/2$

$5C = 1/2$

$C = 1/10$

Particular Intergral : $x = C\,\text{Cos}t + D\,\text{Sin}t$

$= 1/10\,\text{Cos}t + 1/10\,\text{Sin}t$

General Solution

$x = CF + PI$

$= Ae^{-2t} + Be^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$= Ae^{-2t} + Be^{-3t} + \frac{1}{10}[\cos t + \sin t]$

@ $t = 0$, $x = 0.1$ & $dx/dt = 0$

For x: $Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10}[\cos(0) + \sin(0)] = \frac{1}{10}$

$A + B + \frac{1}{10} = \frac{1}{10}$

$A + B = 0$ ——— (1)

For dx/dt: $-2Ae^{-2\cdot 0} - 3Be^{-3\cdot 0} \cancel{+} - \frac{1}{10}\sin[0] + \frac{1}{10}\cos(0) = 0$

$-2A - 3B + \frac{1}{10} = 0$

$-2A - 3B = -\frac{1}{10}$

$2A + 3B = \frac{1}{10}$ ——— (2)

From eqn (1)

$A = -B$ Put into eqn (2)

$-2B + 3B = \frac{1}{10}$

$\therefore B = \frac{1}{10}$

$A = -\frac{1}{10}$

$x = -\frac{1}{10}e^{-2t} + \frac{1}{10}e^{-3t} + \frac{1}{10}[\cos t + \sin t]$