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COMPUTER ENGINEERING

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SOLUTION.

$$\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t$$

Auxiliary equation $= m + 5m + 6$

$m + 5m + 6$

$(m+2)(m+3)$

$m_1 = -2,\ m_2 = -3$

C.F: $x = Ae^{-2} + Be^{-3x}$

P.I: $\Rightarrow x = C\cos t + D\sin t$

$$\frac{dy}{dt} = -C\sin t + D\cos t$$

$$\frac{d^2y}{dt^2} = -C\cos t - D\sin t$$

Sub $\frac{dy}{dt}$ and $\frac{d^2y}{dt^2}$ into original equation.

$[-C\cos t - D\sin t] + 5[-C\sin t + D\cos t] + 6[C\cos t + D\sin t] = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

$\cos t[-C + 5D + 6C] + \sin t[-D - 5C + 6D] = \cos t.$

$\cos t[5D + 5C] - \sin t(-5C + 5D] = \cos t$

Comparing coefficient

$\cos t: 5D + 5C = 1$

$\sin t: -5C + 6D = 0$

$10D - 0 = 1$

$10D = 1$

$D = 1/10$

Input $D = 1/10$ in eqn —— (11)

$5D - 5C = 0$

$5\left(\frac{1}{10}\right) - 5C = 0$

$\frac{1}{2} - 5C = 0$

$5C = \frac{1}{2}$

$C = \frac{1}{2} \times \frac{1}{5}$

$C = \frac{1}{10}$

$C = 1/10$, $D = 1/10$ $\therefore$ $P.I = 1/10 \cos t + 1/10 \sin t$

$G.S = C.F + P.I$

$y = Ae^{-2t} + Be^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

Using the values of $t = 0$, $x = 0.1$, $\frac{dx}{dt} = 0$

$y = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10}\cos(0) + \frac{1}{10}\sin(0)$

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$y = 0.1e^{-2t} - 0.1e^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$y = 0.1e^{-2t} - 0.1e^{-3t} + \frac{1}{10}(\cos t + \sin t)$

$x = k\sin(t + a)$

$\therefore x = \frac{1}{10}\left[(-e^{-2t} + e^{-3t}) + (\cos t + \sin t)\right]$

Using steady part of the equation.

$x = \frac{1}{10}(\cos t + \sin t)$

Note: $dx/dt = 0$ for steady state

$dx/dt = \frac{1}{10}(-\sin t + \cos t) = 0$

$-\sin t + \cos t = 0$

$\cos t = \sin t$

Hence: $t = 45$ or $\pi/4$

$x = \frac{1}{10}(\cos 45 + \sin 45)$

sinusoidal equation: $A\cos\omega t + B\sin\omega t$

$= K\cos(\omega t - \theta)$

but $\cos(\omega t - \theta) = \sin(\omega t - \theta + 90^\circ)$

where $\therefore K = \sqrt{A^2+B^2} = \sqrt{(1/10)^2 + (1/10)^2} = 0.14$

Since it is on the same phase, hence $\theta = 0^\circ$

$x = \frac{1}{10}(\cos 45 + \sin 45)$

$= 0.14\sin(45 + 90^\circ)$

$x = 0.14\sin(45 + 90^\circ)$

$x = 0.14\sin(90 + 45)$

In terms of the above equation.

Hence,

$K = 0.14$ or $\frac{\sqrt{2}}{10}$

$a = 45$ or $\pi/2$

$x = 0.14\sin(t + 45)$