As general solution $\Rightarrow x = Ae^{-2t} + Be^{-3t}$ with $x = 0.1$, $t = 0$ & $\frac{dx}{dt} = 0$

$\Rightarrow 0.1 = Ae^{-2(0)} + Be^{-3(0)}$

$0.1 = Ae^{0} + Be^{0}$

$0.1 = A + B$ —— (1)

$A = -B + 0.1$ —— (2)

$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t}$

$0 = -2Ae^{-2(0)} - 3Be^{-3(0)}$

$0 = -2Ae^{0} - 3Be^{0}$

$0 = -2A - 3B$ —— (3)

putting eqn (2) into eqn (3)

$0 = -2(-B + 0.1) - 3B$

$0 = 2B - 0.2 - 3B$

$0 = -1B - 0.2$

$B = -0.2$

putting B into eqn (1)

$0.1 = A - 0.2$

$A = 0.2 + 0.1$

$0 = \frac{1}{10}(2e^{-2t} - 3e^{-3t} - \sin t + \cos t)$

$0 = 2e^{-2t} - 3e^{-3t} - \sin t + \cos t$

$\sin t - \cos t = 2e^{-2t} - 3e^{-3t}$ — ①

Showing the relationship between trigonometry and exponential

$\sin t = \frac{1}{2m}(e^{-2t} + e^{-3t})$

$\cos t = \frac{1}{2m}(e^{-2t} - e^{-3t})$

Substitute into eqn ①

$\frac{1}{2m}(e^{-2t} + e^{-3t}) - \frac{1}{2m}(e^{-2t} - e^{-3t})$

$e^{-2t}\frac{1}{2m} + e^{-3t}\frac{1}{2m} - \frac{1}{2m}e^{-2t} - \frac{1}{2m}e^{-3t}$

$\sin t - \cos t = 0$

$\sin t = \cos t.$

Divide through by $\cos t$

$\frac{\sin t}{\cos t} = \frac{\cancel{\cos t}}{\cancel{\cos t}}$

$\tan t = 1$

$t = \tan^{-1}(1)$

$t = 45^{0}$

NB: The graph is sinusoidal it is a trig. graph.

$\Rightarrow$ (i.e all exp are zero)

$x = (\cos t + \sin t)\frac{1}{10}$

$x = (\cos 45^{0} + \sin 45^{0})\frac{1}{10}.$

$x = \sqrt{2}$

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Continuation

where
$$k = \sqrt{A^2 + B^2} = \sqrt{\left(\tfrac{1}{10}\right)^2 + \left(\tfrac{1}{10}\right)^2} = \sqrt{\frac{1+1}{(10)^2}}$$

$$k = \frac{\sqrt{2}}{10} \quad //$$

$\theta = 0^0$ because its in same phase.

Recall; $x = k \sin(t + \alpha)$

$$\frac{\sqrt{2}}{10} = \frac{\sqrt{2}}{10} \sin(45 + \alpha).$$

$$1 = \sin(45 + \alpha)$$

$$45 + \alpha = \sin^{-1}(1) = 90$$

$$\alpha = 90 - 45 = 45^0 \Rightarrow \pi/4$$

$\therefore$ The steady state solution is;

$$x = \frac{\sqrt{2}}{10} \sin(t + \pi/4) \quad //$$

HALIM NOBERT UDOCHUKWUKA
16/ENG02/021
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1) $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$ ——— ①

when $x = 0$, $t = 0$ and $\frac{dx}{dt} = 0$,

using $y = e^{kx}$

$x = e^{kt}$

$\frac{dx}{dt} = ke^{kt} = kx$

$\frac{d^2x}{dt^2} = k^2e^{kt} = k^2x$

$k^2x + 5kx + 6kx = 0$

$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$k = \frac{-5 \pm \sqrt{25 - 4(6)}}{2(1)}$

$k = \frac{-5 \pm \sqrt{25 - 24}}{2}$

$k = \frac{-5 \pm \sqrt{1}}{2}$

$k = \frac{-5+1}{2}$ or $\frac{-5-1}{2}$

$k = \frac{-4}{2}$ or $\frac{-6}{2}$

$k = -2$ or -3

$x = \boxed{Ae^{-2t} + Be^{-3t}} \Rightarrow$ General solution.

$P_I^0 = C\cos t + D\sin t$

P_I^{0} :.

$x = C\cos t + D\sin t$

$\frac{dx}{dt} = -C\sin t + D\cos t.$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t.$

Substituting the derivative in ①.

$(-C\cos t - D\sin t) + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) =$

$\cos t - C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D$

$\sin t = \cos t.$

$6C\cos t + 5D\cos t - C\cos t + 6D\sin t - 5C\sin t - D\sin t =$

$\cos t.$

$(6C + 5D - C)\cos t + (6D - 5C - D)\sin t = \cos t.$

$\therefore 6C + 5D - C = 1 \;,\; 6D - 5C - D = 0$

$\cancel{10D}\; 5C + 5D = 1 \;,\; 5D - 5C = 0$

$5C = 5D \;,\; 5D = 5C$

as $D = C$ $\quad D = \frac{5C}{5}$

$5D = -5D + 1$

$5D + 5D = 1 \quad D = C$ //

$10D = 1$

$D = \frac{1}{10}$

$\Rightarrow C = D = \frac{1}{10}$ //

Substituting C & C = D into P.I.

$x = \frac{1}{10}\cos t + \frac{1}{10}\sin t.$

$\Rightarrow x = \frac{1}{10}(\cos t + \sin t)$ //