$\frac{dx}{dt} = -A\sin t + B\cos t$

$\frac{d^2x}{dt^2} = -A\cos t - B\sin t$

$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

$(-A\cos t - B\sin t + 5(-A\sin t + B\cos t) + 6(A\cos t + B\sin t) = \cos t$

$-A\cos t - B\sin t - 5A\sin t + 5B\cos t + 6A\cos t + 6B\sin t = \cos t$

$-A\cos t + 5B\cos t + 6A\cos t - B\sin t - 5A\sin t + 6B\sin t = \cos t$

$\cos t(-A + 5B + 6A) + \sin t(-B - 5A + 6B) = \cos t$

$\cos t(5A + 5B) + \sin t(-5A + 5B) = \cos t$

$5A + 5B = 1 \quad \cdots (1)$

$-5A + 5B = 0 \quad \cdots (2)$

$5B = 5A$

$B = A$

$5(B) + 5(B) = 1$

$10B = 1$

$B = \frac{1}{10}$ since $A = B$

$A = \frac{1}{10}$

P.I. - Partial integration

$x = \frac{1}{10}\cos t + \frac{1}{10}\sin t$

General solution = C.F + P.I

$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - \frac{\sin t}{10} + \frac{\cos t}{10}$

$x = 0.1, \quad dx/dt = 0; \ t = 0$

$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10}\cos(0) + \frac{1}{10}\sin(0)$

$0.1 = A + B + \frac{1}{10} + 0$

$0.1 - \frac{1}{10} = A + B$

$0.1 - 0.1 = A + B$

$0 = A + B$ *

Also,

$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} - \sin(0) + \cos(0)$

$\frac{}{10} \quad \frac{}{10}$

$0 = 2A - 3B + \frac{1}{10}$

$-\frac{1}{10} = -2A - 3B$ **

Solving eqn * and eqn ** simultaneously

$A + B = 0$

$-2A - 3B = -\frac{1}{10}$

$A = -B$

$-2(-B) - 3B = -\frac{1}{10}$

$-2(B) - 3B = -\frac{1}{10}$

$-B = -\frac{1}{10}$; $B = \frac{1}{10}$

$A = -\frac{1}{10}$

$x = -\frac{1}{10}e^{-2t} + \frac{1}{10}e^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

8 Write the steady state solution of the system in the term of

$x = k\sin(t + a)$

$x = 0.1e^{-2t} + 0.1e^{-3t} + 0.1\{\sin t + \cos t\}$

at steady state

$t \to \infty$

$0.1e^{-3t} \to 0$ and $0.1e^{-3t} \to 0$ at steady state

$x = 0.1\{\sin t + \cos t\} \Rightarrow 0.1\sin t + 0.1\cos t$

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ENG 381. Engineering Mathematics III

1) The dynamic model of a body in motion,

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

where $t=0$, $x = 0.1$ and $\frac{dx}{dt} = 0$

solution

If $\frac{d^2y}{dx^2} + a\frac{dy}{dx} + cy = 0$

then $y = x$ and $x = t$

Hence

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Assume $x = Ae^{kt}$

$k^2x + 5kx + 6x = 0$ —— Auxiliary equation

$k^2 + 5k + 6 = 0$

$k^2 + 3k + 2k + 6 = 0$

$k(k+3) + 2(k+3) = 0$

$(k+2)(k+3) = 0$

$k = -2$ or $k = -3$

General solution

$x = Ae^{k_1 t} + Be^{k_2 t}$

$x = Ae^{-2t} + Be^{-3t}$

for $\cos t$

$x = \cos t$

$x = A\cos t + B\sin t$

$x = k\sin(t+a)$

$\Rightarrow 0.1\sin t + 0.1\cos t$

$x = k\sin(t+a)$

$\Rightarrow k\sin t\cos a + k\sin a\cos t$

Comparing (1) and (2)

$k\sin a = 0.1$ —— (ii)

$k\cos a = 0.1$ —— (iii)

$k^2\sin^2 a + k^2\cos^2 a = 0.1^2 + 0.1^2$

$k^2\sin^2 a + \cos^2 a = 0.02$

$k^2 = 0.02$

$k = 0.1414$

$\Rightarrow k\sin a = k\cos a = 0.1$

$k\sin a = k\cos a$

$\sin a = \cos a$

$\frac{\sin a}{\cos a} = 1$

$\tan a = 1$

$a = \tan^{-1} 1$

$a = 45^\circ$

$\Rightarrow x = 0.1\{\sin t + \cos t\}$

$x = 0.1414\sin(t + 45^\circ)$