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- PETROLEUM ENGINEERING
- ENG 381 ASSIGNMENT 1

### Solution to the Assignment

$$I. \quad \frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Using the Auxiliary Equation method to obtain the solution of the model.

a) Finding the Complementary Function, CF

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$$

$$m^2 + 5m + 6 = 0$$

$$m^2 + 2m + 3m + 6 = 0$$

$$m(m+2) + 3(m+2) = 0$$

$$(m+2)(m+3) = 0$$

$$m+2 = 0 \text{ or } m+3 = 0$$

$$m = -2 \text{ or } m = -3$$

$$m_1 = -2 \text{ and } m_2 = -3$$

Therefore, the CF is  $x = Ae^{-2t} + Be^{-3t}$  (\*)

b) For the Particular Integral

$$f(t) = \cos t$$

$$x = C \cos t + D \sin t \quad (**)$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

Substitution into the Original equation

$$-C \cos t - D \sin t + 5[-C \sin t + D \cos t] + 6[C \cos t + D \sin t] = \cos t$$

$$-C \cos t - D \sin t - 5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t = \cos t$$

$$5D \cos t - C \cos t + 6C \cos t + 6D \sin t - D \sin t - 5C \sin t = \cos t$$

$$5D \cos t + 5C \cos t + 5D \sin t - 5C \sin t = \cos t$$

$$\cos t [5D + 5C] + \sin t [5D - 5C] = \cos t$$

Equating coefficients

$$5D + 5C = 1 \quad (\text{i})$$

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$$5D - 5C = 0 \quad (\text{ii}) \quad \text{solving by Elimination Method}$$

$$5C - (-5C) = 1 - 0$$

$$5C + 5C = 1$$

$$10C = 1$$

$$C = \frac{1}{10}$$

Substitute into equation(i)

$$5D + 5\left[\frac{1}{10}\right] = 1$$

$$5D + \frac{1}{2} = 1 \quad (\text{ii})$$

$$5D = 1 - \frac{1}{2}$$

$$5D = \frac{1}{2}$$

$$10D = 1$$

$$D = \frac{1}{10}$$

Therefore, the Particular Integral, PI is  $x = \frac{\cos t}{10} + \frac{\sin t}{10}$  (\*\*\*)

Complete general solution = CF + PI

Therefore, General Solution is  $x = Ae^{-2t} + Be^{-3t} + \frac{\cos t}{10} + \frac{\sin t}{10}$  (\*\*\*\*)

$$\text{At } t = 0, x = 0.1 \text{ and } \frac{dx}{dt} = 0$$

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{\cos(0)}{10} + \frac{\sin(0)}{10}$$

$$0.1 = A + B + \frac{1}{10} + 0$$

$$A + B = \frac{1}{10} - 0.1$$

$$A + B = 0 \quad (\text{iii})$$

From equation (\*\*\*\*)

$$x = Ae^{-2t} + Be^{-3t} + \frac{\cos t}{10} + \frac{\sin t}{10}$$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - \frac{\sin t}{10} + \frac{\cos t}{10}$$

$$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} - \frac{\sin(0)}{10} + \frac{\cos(0)}{10}$$

$$0 = -2A - 3B - 0 + \frac{1}{10}$$

$$2A + 3B = \frac{1}{10} \quad (\text{iv})$$

From equation (iii)

$$A + B = 0$$

$$A = -B \quad (\text{v})$$

Substitute equation (v) into (iv)

$$2(-B) + 3B = \frac{1}{10}$$

$$-2B + 3B = \frac{1}{10}$$

$$B = \frac{1}{10}$$

Substitute for B into (iii)

$$A + \left[ \frac{1}{10} \right] = 0$$

$$A = -\frac{1}{10}$$

The Particular Solution for the model is  $x = \frac{-e^{-2t}}{10} + \frac{e^{-3t}}{10} + \frac{\cos t}{10} + \frac{\sin t}{10}$

$$x = \frac{1}{10} [e^{-3t} - e^{-2t} + \cos t + \sin t]$$

III. To write the Steady state solution in the form of  $x = K \sin(t + a)$

The steady state solution of the body is the Particular Integral on the RHS of the General Solution.

From Equation (\*\*\*\*)

The steady state solution is  $x = \frac{1}{10} [\cos t + \sin t]$

Because the LHS;  $\frac{1}{10} [e^{-3t} - e^{-2t}]$  tends to infinite. This is known as the Transient solution.

$$x = \frac{1}{10} [\cos t + \sin t]$$

substituting  $\frac{\sqrt{2}}{\sqrt{2}}$  which is equal to 1

$$x = \frac{1}{10} \left[ \cos t \frac{\sqrt{2}}{\sqrt{2}} + \sin t \frac{\sqrt{2}}{\sqrt{2}} \right]$$

$$x = \frac{\sqrt{2}}{10} \left[ \cos t \frac{1}{\sqrt{2}} + \sin t \frac{1}{\sqrt{2}} \right]$$

Recall that  $\sin(45^\circ) = \cos(45^\circ) = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$$x = \frac{\sqrt{2}}{10} [\cos t \sin(45) + \sin t \cos(45)]$$

$$x = \frac{\sqrt{2}}{10} [\sin(t + 45^\circ)]$$

{Hint (3) :  $\sin(a + b) = \cos a \sin b + \sin a \cos b$ }

Recall that  $\frac{\pi}{4} = 45^\circ$

So substituting  $\frac{\pi}{4} = 45$

$$\therefore x = \frac{\sqrt{2}}{10} \left[ \sin\left(t + \frac{\pi}{4}\right) \right] \dots \quad (***)$$

So equation (\*\*\*\*\*) is the steady state solution in the form  $x = K \sin(t + a)$ .

$$\text{Where } K = \frac{\sqrt{2}}{10}$$

$$a = \frac{\pi}{4}$$