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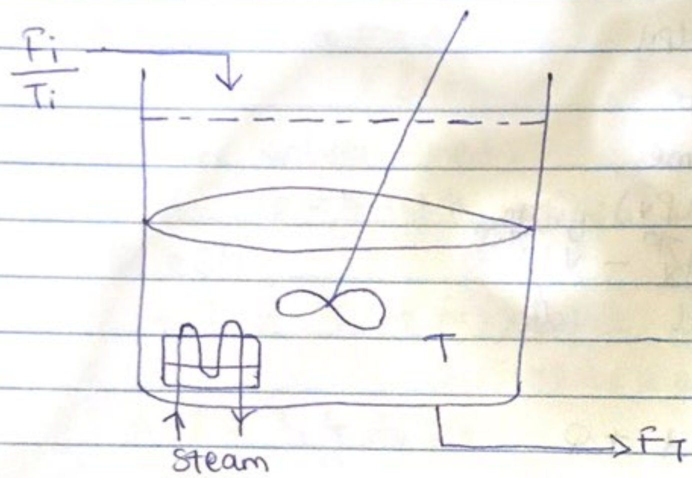
Course code: CHE 531

Course title: Process Dynamics & Control I

Assignment 1



using energy balance

$$[U + PE + KE]_{t+\Delta t} - [U + PE + KE]_t = \left[\underset{\substack{\text{into the system} \\ \text{due to convection}}}{H + PE + KE}\right]_{\Delta t} - \left[\underset{\text{out of system}}{H + PE + K}\right.$$

$$+ Q - W_s$$

where W_s = shaft work

PE = Potential energy

KE = kinetic energy

Q = heat

Assumptions

$W_s = 0$ Because the tank is steady

$PE = 0$

$KE = 0$ Because there is no elevation of the product in the system.

$[U_{sys}]_{t+\Delta t} - [U_{sys}]_t = \dot{H}_{in} \Delta t - \dot{H}_{out} \Delta t + Q \Delta t$

Dividing through by Δt and limiting $t \to 0$

$$\frac{dU}{dt} = \dot{H}_{in} - \dot{H}_{out} + Q$$

from first Law of thermodynamics

$H = U + PV$

$\therefore U = H - PV$

where H = Enthalpy

P = Pressure

V = volume

$U_{sys} = H_{sys} - (PV)_{system}$

$$\frac{dU}{dt} = \frac{dH}{dt} - P\cancel{\frac{dV}{dt}}^{0} - V\cancel{\frac{dP}{dt}}^{0}$$

Substitute dU/dt

$$\frac{dH}{dt} = \dot{H}_{in} - \dot{H}_{out} + Q$$

$H = \dot{m}h$

$m = \rho V \quad \dot{m} = \rho F$

Replacing $H = \dot{m}h$

$$\frac{d(m_{sys}h)}{dt} = (\dot{m}h)_{in} - (\dot{m}h)_{out} + Q$$

Replace $m = \rho V$ & $\dot{m} = \rho F$

$$\frac{d(\rho V h)}{dt} = \rho F h_{in} - \rho F h_{out} + Q$$

$$\rho V \frac{dh}{dt} = \rho F h_{in} - \rho F h_{out} + Q$$

from the question $Q = \dot{m}_s \lambda_s$, $h = C_p dT$

Recall C_p is constant with time.

$$\rho V C_p \frac{dT}{dt} = F\rho C_p \left[\int_{T_0}^{T_i} dT\right]_{in} - F\rho C_p \left[\int_{T_0}^{T} dT\right]_{out} + \dot{m}_s \lambda_s$$

$$\rho V C_p \frac{dT}{dt} = F\rho C_p [T_i - T_0]_{in} - F\rho C_p [T - T_0]_{out} + \dot{m}_s \lambda_s$$

Divide through by $F \rho C_p$

$$\frac{\rho u C_p dT}{F \rho C_p dt} = \frac{F \rho C_p (T_1 - T_0)}{F \rho C_p} - \frac{F \rho C_p (T - T_0)}{F \rho C_p} + \frac{\dot{m}_s \lambda_s}{F \rho C_p}$$

$$\frac{V}{F}\frac{dT}{dt} = [T_1 - T_0] - [T - T_0] + \frac{\dot{m}_s \lambda_s}{F \rho C_p}$$

$$\frac{V}{F}\frac{dT}{dt} = T_1 - T + \frac{\dot{m}_s \lambda_s}{F \rho C_p}$$

$$\frac{V}{F}\frac{dT}{dt} + T = T_1 + \frac{\dot{m}_s \lambda_s}{F \rho C_p}$$

Substituting the values where:

$\rho = 1000 \, kg/m^3$, $C_p = 4.181 \, KJ/kg°c$, $F_i = 0.15 \, m^3/min$ $V = 3$

$\lambda_s = 2258 \, KJ/kg$

$$\therefore \frac{3}{0.15}\frac{dT}{dt} + T = T_1 + \frac{\dot{m}_s \times 2258}{0.15 \times 1000 \times 4.181}$$

$$2.0 \frac{dT}{dt} + T = T_1 + \dot{m}_s 3.6$$

Deviation variable = Dynamic model − Steady state models

$$\therefore 2.0 \frac{dT'}{dt} + T' = T_1' + 3.6 \dot{m}_s \rightarrow \text{Deviation variable}$$

Transfer functions

$$\frac{dT}{dt} = sT(s) - T(0)$$

$$T = T(s)$$

Substitute in the deviation variable

$$2.0 [sT'(s) - T'(0)] + T'(s) = T_1'(s) + 3.6 \dot{m}_s'(s)$$

$$2.0 \, sT'(s) + T'(s) = T_1'(s) + 3.6 \, m's(s)$$

factorize $T'(s)$

$$T'(s)(2.0 + 1) = T_1'(s) + 3.6 \dot{m}'_s(s)$$

$$T'(s) = \frac{T_1'(s)}{2.0s + 1} + \frac{3.6}{2.0s + 1} \times m'_s(s)$$

~~T'(s)~~ $T'(s)[2.0s + 1] = T_1'(s) + 3.6 \dot{m}'s(s)$

making $T'(s)$ the subject of the formular

$$T'(s)\frac{[2.0+1]}{2.0+1} = \frac{T_i'(s)}{2.0s+1} + \frac{3.6\,\dot{m}'s(s)}{2.0s+1}$$

$$T'(s) = \frac{T_i'(s)}{2.0s+1} + \frac{3.6}{2.0s+1} \cdot \dot{m}_s'(s)$$