AGWANIRV ROSEMART ONYINYECHI
CHEMICAL ENGINEERING
17ENG01/1003
ENG 281C ENGINEERING MATHS)

1. The hypothenus of a right-angled triangle is denoted as c and the other two sides are denoted as a and b. If the possible error of measuring each a and b is $\pm 1.5\%$. find the maximum possible error in calculating

a) Area of the triangle

b) The length of the hypothenus.

SOLUTION

a. Let the area of the triangle be A

$A = \frac{1}{2} a \times b$

$\delta A = \frac{\partial A}{\partial a} \cdot \delta a + \frac{\partial A}{\partial b} \cdot \delta b$

$\frac{\partial A}{\partial a} = \frac{b}{2}$ and $\frac{\partial A}{\partial b} = \frac{a}{2}$

$\delta a = \frac{\pm 1.5}{100} \cdot a \quad \delta b = \frac{\pm 1.5 \cdot b}{100}$

$\delta A = \frac{b}{2} \cdot \left(\frac{\pm 1.5 a}{100}\right) + \frac{a}{2}\left(\frac{\pm 1.5 b}{100}\right)$

$\delta A = \pm \frac{ab}{2}\left[\frac{1.5}{100} + \frac{1.5}{100}\right]$

$\delta A = \pm \frac{ab}{2}\left[\frac{3}{100}\right]$

$\delta A = \pm 3\% A$

b. $h = \sqrt{a^2 + b^2} = (a^2 + b^2)^{1/2}$

$\delta h = \frac{\partial h}{\partial a} \cdot \delta a + \frac{\partial h}{\partial b} \cdot \delta b$

$$\frac{\partial h}{\partial a} = \frac{1}{2}(a^2+b^2)^{-1/2}(2a) = \frac{a}{\sqrt{a^2+b^2}}$$

$$\frac{\partial h}{\partial b} = \frac{1}{2}(a^2+b^2)^{-1/2}(2b) = \frac{b}{\sqrt{a^2+b^2}}$$

$$\delta a = \frac{\pm 1.5}{100} a \quad , \delta b = \frac{\pm 1.5}{100} b$$

$$\delta h = \frac{a}{\sqrt{a^2+b^2}} \cdot \left(\frac{\pm 1.5 a}{100}\right) + \frac{b}{\sqrt{a^2+b^2}} \left(\frac{\pm 1.5 b}{100}\right)$$

$$\delta h = \frac{\pm\, a^2+b^2}{\sqrt{a^2+b^2}} \left(\frac{1.5}{100}\right)$$

$$= \frac{\pm 1.5}{100} \sqrt{a^2+b^2} =$$

$$= 0.015\% \; h,$$