OLEOBIBI CLEMENTINA ONYINYECHUKWU

Elect/elect

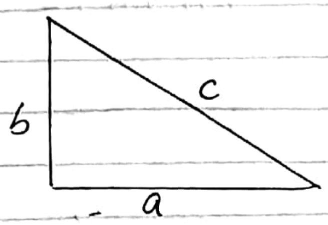
17/ENG04/027 ENG281 Assignment II

The hypotenuse of a right-angled triangle is denoted at c and the other two sides are denoted as a and b. If the possible error of measuring each of a and b is ±1.5%. Find the maximum possible error in calculating

(a) The Area of the triangle.

(b) The length of the hypotenuse.

Soln



(a)

$$Area(A) = \frac{1}{2}ab$$

using $\delta A = \left[\frac{\partial A}{\partial a} \times \delta a\right] + \left[\frac{\partial A}{\partial a} \times \delta b\right]$

$$\frac{\partial A}{\partial a} = \frac{1}{2}b$$

$$\frac{\partial A}{\partial b} = \frac{1}{2}a$$

$$\delta A = \left[\frac{1}{2}b \times \left(\pm\frac{1.5}{100}a\right)\right] + \left[\frac{1}{2}a \times \left(\pm\frac{1.5b}{100}\right)\right]$$

$$\delta A = \left[\frac{1}{2}ba \times \left(\pm\frac{1.5}{100}\right)\right] + \left[\frac{1}{2}ab \times \left(\pm\frac{1.5}{100}\right)\right]$$

$$\delta A = \pm\frac{1}{2}ba \times \left(\frac{1.5}{100}\right) + \pm\frac{1}{2}ab \times \left(\frac{1.5}{100}\right)$$

$$\delta A = \pm\frac{1}{2}ba\left(\frac{1.5}{100} + \frac{1.5}{100}\right)$$

$$\delta A = \pm\frac{1}{2}ba\left(\frac{3}{100}\right)$$ but recall $A = \frac{1}{2}ab$

$$\delta A = \pm A(3\%)$$

$$\therefore \delta A = \pm 3\% A$$

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(b) $c^2 = a^2 + b^2$

$c = \sqrt{a^2+b^2} \Rightarrow c = (a^2+b^2)^{1/2}$

Hence $c = f(a,b)$

$$\delta c = \left[\frac{\partial c}{\partial a} \times \delta a\right] + \left[\frac{\partial c}{\partial b} \times \delta b\right]$$

$$\frac{\partial c}{\partial a} = \frac{1}{\cancel{2}}(a^2+b^2)^{-1/2} \times \cancel{2}a = a(a^2+b^2)^{-1/2} = \frac{a}{(a^2+b^2)^{1/2}}$$

$$\frac{\partial c}{\partial b} = \frac{1}{\cancel{2}}(a^2+b^2)^{-1/2} \times \cancel{2}b = b(a^2+b^2)^{-1/2} = \frac{b}{(a^2+b^2)^{1/2}}$$

$$\delta c = \left[\frac{a}{(a^2+b^2)^{1/2}} \times \left(\pm\frac{1.5a}{100}\right)\right] + \left[\frac{b}{(a^2+b^2)^{1/2}} \times \left(\pm\frac{1.5b}{100}\right)\right]$$

but recall $\Rightarrow h = (a^2+b^2)^{1/2}$

$$\delta c = \left[\frac{a}{h} \times \left[\pm\frac{1.5a}{100}\right]\right] + \left[\frac{b}{h} \times \left(\pm\frac{1.5b}{100}\right)\right]$$

$$\delta c = \left[\frac{a^2}{h} \times \frac{\pm 1.5}{100}\right] + \left[\frac{b^2}{h} \times \frac{\pm 1.5}{100}\right]$$

$$\delta c = \pm\frac{1.5}{100}\left[\frac{a^2}{h} + \frac{b^2}{h}\right]$$

$$\delta C = \pm\frac{1.5}{100} \times \frac{1}{h}\left[a^2+b^2\right]$$

but $h^2 = a^2 + b^2$

$$\delta c = \pm\frac{1.5}{100} \times \frac{1}{\cancel{h}}(h^{\cancel{2}})$$

$$\delta c = \pm 1.5\% h$$

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